MATH 252-3 **Spring 2005**

Vector Calculus

Homework Set 10 Due Wednesday, 6 April 2005

Course Web Site: `http://www.math.sfu.ca/~ralfw/math252/`

Textbook: **Davis and Snider** "*Introduction to Vector Analysis*"

Reading: Sections 4.8–9, 5.1, 5.4–5

Problems to study (for practice; you do not need to hand these in):

- Section 4.8 (pp.256–257): 1, 3, 4, 5
- Section 4.9 (pp.262–265): 3(a), 4, 6, 10, 13, 16, 18, 21, 23, 26, 27, 28, 35
- Section 5.1 (pp.276–277): 1, 2, 3, 4, 5, 10
- Section 5.2 (pp.284–285): 8(b,c)
- Section 5.4 (pp.294–295): 1, 3, 8, 9, 10, 12
- Section 5.5 (pp.299–300): 4, 5, 6, 7

Problems to hand in:

- *Section 4.8 (pp.256–257): 6*
- *Section 4.9 (pp.262–265): 3(b), 5, 8, 12, 17, 20, 29, 30, 34*
- *Section 5.1 (pp.276–277): 6, 7, 8*
- *Section 5.2 (pp.284–285): 8(a), 9*
- *Section 5.4 (pp.294–295): 2, 4, 6, 7, 11*
- *Section 5.5 (pp.299–300): 1, 2, 3*

Notes:

1. The (optional) problems 5–7 of Section 5.5 should help you solidify your understanding of the Divergence Theorem and Stokes' Theorem, and you are encouraged to read and think about these.

Extra problem on reverse...

Extra problem:

1. *Vector-valued Integrals*

   Surface and volume integrals of vector-valued functions may be defined as limits of Riemann sums, in a similar way to the definition of integrals of scalar-valued functions; alternatively, one may consider them componentwise; for instance for $\mathbf{G} = G_1\mathbf{i} + G_2\mathbf{j} + G_3\mathbf{k}$, we can define the volume integral as

   $$\iiint_V \mathbf{G}\, dV = \mathbf{i}\iiint_V G_1\, dV + \mathbf{j}\iiint_V G_2\, dV + \mathbf{k}\iiint_V G_3\, dV.$$

   We can use the divergence theorem, or the corresponding results for components in the form

   $$\iiint_V \frac{\partial G_1}{\partial x}\, dV = \iint_S G_1 \mathbf{i}\cdot\mathbf{n}\, dS\ , \qquad \text{(and similarly for } y,\ z \text{ components)}$$

   to obtain identities for such vector-valued integrals (where S is the surface of the region V, and $\mathbf{n}$ is the outward unit normal).

   (a) Let f be a scalar field. Show the identity

   $$\iiint_V \nabla f\, dV = \iint_S f\mathbf{n}\, dS\ \left(= \iint_S f\, d\mathbf{S} \right)\ ,$$

   and hence deduce a vector interpretation of the limit

   $$\lim_{V\to\mathbf{0}} \frac{\iint_S f\mathbf{n}\, dS}{V}\ .$$

   [You should use two different methods to show the first identity: (a) Establish the result componentwise; (b) Apply the divergence theorem to the vector field $\mathbf{G} = f\mathbf{C}$, where $\mathbf{C}$ is an arbitrary *constant* vector.]

   (b) Let $\mathbf{F}$ be a vector field. Show the identity

   $$\iiint_V \nabla\times\mathbf{F}\, dV = \iint_S \mathbf{n}\times\mathbf{F}\, dS,$$

   and hence give a vector interpretation of the limit

   $$\lim_{V\to\mathbf{0}} \frac{\iint_S \mathbf{n}\times\mathbf{F}\, dS}{V}\ .$$