

## Vector Calculus

Homework Set 2

Due Wednesday, 26 January 2005

Course Web Site: <http://www.math.sfu.ca/~ralfw/math252/>Textbook: **Davis and Snider** “*Introduction to Vector Analysis*”:

Reading: Sections 1.8 – 1.15

Problems to study (for practice; you do not need to hand these in):

- Section 1.8 (pp.29–30): 4, 8, 15
- Section 1.10 (pp.38–40): 5, 8, 19, 22, 27
- Section 1.12 (pp.51–53): 18, 24, 26
- Section 1.13 (pp.57–59): 5, 7, 10, 16, 19
- Section 1.14 (pp.60–61): **3**, 5, 10, 11

Problems to hand in:

- Section 1.8 (pp.29–30): 7, 11, 12, 18
- Section 1.10 (pp.38–40): 9, 12, 18, 28
- Section 1.12 (pp.51–53): 27
- Section 1.13 (pp.57–59): 3, 11, 15, 17
- Section 1.14 (pp.60–61): **2**, 6, **7**, 9
- Section 1.15 (p.66): 2

For problems **2** and **7** of section 1.14 (and the optional problem **3**), please derive the relevant identities *both* (i) using previously derived identities and properties, using the methods of section 1.14, and (ii) using the tensor notation conventions and methods of section 1.15. Also show problem **7** using the geometric definitions of the dot and cross products.

Notes:

- For section 1.8 problem 18, either find the point(s) of intersection of the lines, or show that none exist.
- Observe that section 1.13 problem 17 gives the expansion of a vector  $\mathbf{V}$  in terms of any three *independent* vectors  $\mathbf{A}$ ,  $\mathbf{B}$  and  $\mathbf{C}$  (they don't need to be orthogonal).
- Recall the Cauchy-Schwarz inequality

$$|\mathbf{A} \cdot \mathbf{B}| \leq |\mathbf{A}||\mathbf{B}| \implies |\mathbf{A}|^2|\mathbf{B}|^2 - (\mathbf{A} \cdot \mathbf{B})^2 \geq 0.$$

The result of section 1.14 problem 7,  $|\mathbf{A} \times \mathbf{B}|^2 = |\mathbf{A}|^2|\mathbf{B}|^2 - (\mathbf{A} \cdot \mathbf{B})^2$ , provides a formula for the magnitude of the “error” in the Cauchy-Schwarz inequality. It also shows that the Cauchy-Schwarz inequality is an equality exactly when  $\mathbf{A} \times \mathbf{B} = \mathbf{0}$ , that is, if  $\mathbf{A} = \mathbf{0}$  or  $\mathbf{B} = \mathbf{0}$  or  $\mathbf{A}$  and  $\mathbf{B}$  are parallel.