MATH 252-3 **Spring 2005**

Vector Calculus

Homework Set 5 Due Wednesday, 16 February 2005

Course Web Site: `http://www.math.sfu.ca/~ralfw/math252/`

Textbook: **Davis and Snider** "*Introduction to Vector Analysis*"

Reading: Sections 3.2–3.5

Problems to study (for practice; you do not need to hand these in):

- Section 3.2 (p.117): 1, 2
- Section 3.3 (pp.124–125): 3, 5, 9
- Section 3.4 (p.132): 2, 6, 10, 13

Problems to hand in:

- *Section 3.2 (p.117): 3, 4*
- *Section 3.3 (pp.124–125): 4, 6, 7, 8, 12*
- *Section 3.4 (p.132): 4, 7, 9, 11, 12*
- *Section 3.5 (pp.135–136): 10 (relate your answer to that of Section 3.4 problem 9)*

Extra problem (to hand in)

1. Consider the scalar field $f(x,y,z) = x^2 + y^2 - z^2$.

   (a) Plot in Maple the isotimic (level) surfaces given by $f(x,y,z) = x^2 + y^2 - z^2 = C^2$ for $C = 1$, 2 and 3.

   (b) Compute the gradient field $\mathbf{grad} f$, and plot this gradient field in Maple (see the `fieldplot3d` and `gradplot3d` commands in the `fields.mws` or `fields_nf.mws` sample code).

   (c) What are the general equations of the flow lines through this gradient field? Write down the equations for the flow lines through the points (1,1,1), (1,1,2) and (1,1,3), and plot these three flow lines in Maple, together with the isotimic surfaces on the same graph.

Some Notes on page 2. . .

Notes

1. Note that in problem 4 of Section 3.3 (p.124) you are computing the divergence of the field

$$\mathbf{F} = \frac{x\mathbf{i} + y\mathbf{j} + z\mathbf{k}}{(x^2+y^2+z^2)^{3/2}} = \frac{\mathbf{R}}{R^3} = \frac{1}{R^2}\hat{\mathbf{R}},$$

which is (up to constants) the vector field corresponding to an inverse square force law, such as the gravitational or electrostatic field. The result you should obtain is $div\ \mathbf{F} = 0$ for $R \neq 0$ i.e. $\mathbf{R} \neq \mathbf{0}$; you can interpret the result as saying, for instance, that the electrostatic field generated by a point charge is divergence-free away from the charge.

2. Problem 12 of Section 3.3 (p.124) gives the mathematical statement of a property that was mentioned in class: The divergence measures the fractional rate of change of volume.

3. Problem 12 of Section 3.4 (p.132) has the result that $\mathbf{curl}(f(R)\mathbf{R}) = \mathbf{0}$ for any differentiable function f. This is clear from the interpretation of the curl: a purely radial vector field has zero circulation around any curve, or (maybe more obviously) a vector field pointing purely in a radial direction cannot cause any paddle wheel to rotate.