MATH 467-3 **Fall 2003**

Dynamical Systems

Homework Set 0 Due Wednesday, 10 September 2003

Course Web Site: `http://www.math.sfu.ca/~ralfw/math467/`

Welcome to MATH 467! This "Homework Set 0" is a quick review, with the main goal to encourage you to use software such as Matlab or Maple in addition to your analytical "pencil-and-paper" work. Furthermore, I would like you to think about the course a little, and look forward to learning a bit about your interests and current views on dynamical systems. This set is not "required"; work received by Wednesday, September 10 will be eligible for up to 30% bonus on a homework set.

1. This problem is intended as a review and warm-up for our study of one-dimensional dynamical systems.
   Consider the differential equation
   $$\dot{x} = rx - x^2, \tag{1}$$
   where $r > 0$.

   (a) Derive the general solution analytically. [Hint: use partial fractions...]

   (b) Find the particular solution with $x(0) = x_0$.

   (c) If $x_0 > 0$, deduce the long-time behaviour, that is, the value of $\lim_{t\to\infty} x(t)$.

   (d) Could you have obtained the answer to (c) without deriving the complete analytical solution? To do this, you may wish to find all fixed points of equation (1), and determine their stability.

   For the remaining parts, assume $r = 3$ and $x_0 = 1$.

   (e) Use computer software such as Maple or Matlab to plot your solution from (b) for $0 \le t \le 5$. (You might also wish to compare your solutions to (a) and (b) above with an analytical solution obtained using Maple.)

   (f) Now solve the equation numerically (without using the analytical solution from (b)): Using a numerical differential equation solver, solve the ODE
   $$\dot{x} = 3x - x^2, \qquad x(0) = 1$$
   directly for $0 \le t \le 5$, plot the solution and compare the resulting plot with that obtained in (e).
   [Possible numerical ODE solvers are:
   - Matlab: `ode45`
   - Maple: `DEplot`, or `dsolve(..., type=numeric)` with `odeplot`]

2. I would like you to write a few paragraphs (not more than a page) on some aspect or application of dynamical systems or differential equations you might find interesting or would like to learn more about; or you could give your (current) answer to the question "why are nonlinear dynamical systems interesting/scientifically important?". This could be based on something you learnt in Math 310, or other mathematics or science classes, something you have read elsewhere or found on the web. Please write your paragraphs neatly (preferably typed), proofread and give references as needed. I would like to learn what you find interesting, and look forward to seeing what you come up with!