Parallel immersed boundary simulations of worm-like swimmers in the inertial flow regime

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Introduction

- Hydrodynamic interactions in suspensions of swimming organisms has been a topic of recent interest.
- Theoretical approaches to understanding the collective swimming have followed two paths, microscopic and coarse-grained approach, here we focus on the former.
- We are interested in flows at moderate Reynolds numbers in the range from 0.1 to 100, in which both advective and diffusive effects are present.

Figure 1: Suspension of Nematodes

Immersed Boundary Method

- We have two separate meshes, one on the fluid domain and mesh on the structure.
- Immersed boundary method, enables us to represent the fluid on an Eulerian grid and the boundary within the fluid, immersed boundary, on a Lagrangian grid.
- The Eulerian and Lagrangian variables are then linked by a Dirac delta distribution.

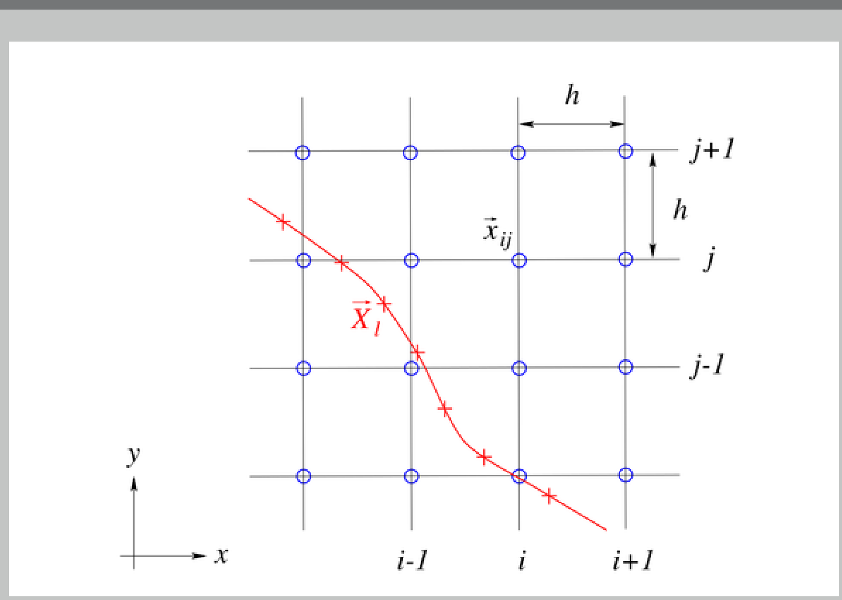

Figure 2: **O**: Immersed Boundary, **+**: Fluid gridpoints

Mathematical Formulation

- IB Formulation: Motion of the fluid is governed by incompressible Navier-Stokes equations:

$$\rho\frac{\partial \mathbf{u}}{\partial \mathbf{t}} + \rho \mathbf{u}\cdot\nabla\mathbf{u} = \mu\Delta\mathbf{u} - \nabla \mathbf{p} + \mathbf{F}, \tag{1}$$

$$\nabla\cdot\mathbf{u} = 0, \tag{2}$$

- External force **F** is projected onto the fluid domain using delta function.

$$\mathbf{F}(\mathbf{x},\mathbf{t}) = \int_\Gamma \mathbf{f}(\mathbf{s},\mathbf{t})\delta(\mathbf{x}-\chi(\mathbf{s},\mathbf{t}))\mathbf{ds}. \tag{3}$$

- We need another equation to close above system, which is given by a no-slip boundary condition on the immersed boundary points, i.e.

$$\frac{\partial\chi}{\partial \mathbf{t}} = \mathbf{u}(\chi(\mathbf{s},\mathbf{t}),\mathbf{t}) = \int_\Omega \mathbf{u}(\mathbf{x},\mathbf{t})\delta(\mathbf{x}-\chi(\mathbf{s},\mathbf{t}))\mathbf{dx}, \tag{4}$$

where χ is position of swimmer(s), and **s** which is parameterized by **s**.

- Force Density: Accounting for the two types of forces, elastic and bending forces, energy is defined:

$$\mathbf{E} = \int_\Gamma \sigma_e\left(\left|\frac{\partial\chi}{\partial \mathbf{s}}\right|;\mathbf{s}\right)\mathbf{ds} + \frac{1}{2}\int_\Gamma \sigma_b\left|\frac{\partial^2\chi}{\partial \mathbf{s}^2} - \frac{\partial^2\tilde{\chi}}{\partial \mathbf{s}^2}\right|\mathbf{ds}, \tag{5}$$

L is the resting length of organism, and $\tilde{\chi}$ is the target position of the organism. The resulting force is therefore

$$\mathbf{F} = -\nabla\mathbf{E}. \tag{6}$$

$$\mathbf{F} = \sigma_e\frac{\partial}{\partial \mathbf{s}}\left(\frac{\partial\chi}{\partial\mathbf{s}}\left(1-\frac{\mathbf{L}}{\left|\frac{\partial\chi}{\partial\mathbf{s}}\right|}\right)\right) + \sigma_b\frac{\partial^2}{\partial\mathbf{s}^2}\left(\frac{\partial^2\chi}{\partial\mathbf{s}^2} - \frac{\partial^2\tilde{\chi}}{\partial\mathbf{s}^2}\right), \tag{7}$$

- Let the target curvature $\mathbf{c}(\mathbf{s},\mathbf{t}) = \frac{\partial^2\tilde{\chi}}{\partial\mathbf{s}^2}$ and $\tilde{\chi}(\mathbf{s},\mathbf{t}) = (\mathbf{s}, \mathbf{a}\sin(\kappa\mathbf{s}-\omega\mathbf{t}))$

$$\mathbf{c}(\mathbf{s},\mathbf{t}) = (0, -\kappa^2\mathbf{a}\sin(\kappa\mathbf{s}-\omega\mathbf{t})) \tag{8}$$

Numerical Method

- Fractional step IB algorithm
  Step IB1: Evolve the IB position to time $\mathbf{t}_{\mathbf{n}+1/2} = (\mathbf{n}+1/2)\mathbf{t}$
  Step IB2: Calculate the fluid forcing term
  Step IB3: Solve the incompressible NavierStokes equations

- Parallel Implementation
  The parallelization is performed by subdividing the rectangular domain $\mathbf{\Omega}$into equally sized rectangular partitions $\mathbf{\Omega}_{\mathbf{I},\mathbf{J},\mathbf{K}}$.Each node is allocated a single domain partition $\mathbf{\Omega}_{\mathbf{I},\mathbf{J},\mathbf{K}}$, along with the values of the Eulerian and Lagrangian variables contained within it.

$\Omega_{1,3}$	$\Omega_{2,3}$	$\Omega_{3,3}$
$\Omega_{1,2}$	$\Omega_{2,2}$	$\Omega_{3,2}$
$\Omega_{1,1}$	$\Omega_{2,1}$	$\Omega_{3,1}$

Figure 3: Parallel domain decomposition

Parameters Choice

- Paramter choice for single swimmer

Density	Viscosity	Nd	Ns	Δt	L	Ls	a
1 g/cm^3	**1 cP**	64	128	$\mathbf{O(10^{-5})s}$	0.4 cm	0.1cm	Ls/10

Table 1: Parameters

Parameters Choice

- Reynolds number

$$\mathbf{Re} = \frac{\rho\mathbf{LV}}{\mu} = \frac{\rho\omega}{\mu\kappa^2}$$

Simulations: a. swimming speed

- We simulated single organisms, to examine swimming speed variations for different Reynolds numbers. Next, we invstigated the dynamics of double-swimmers.

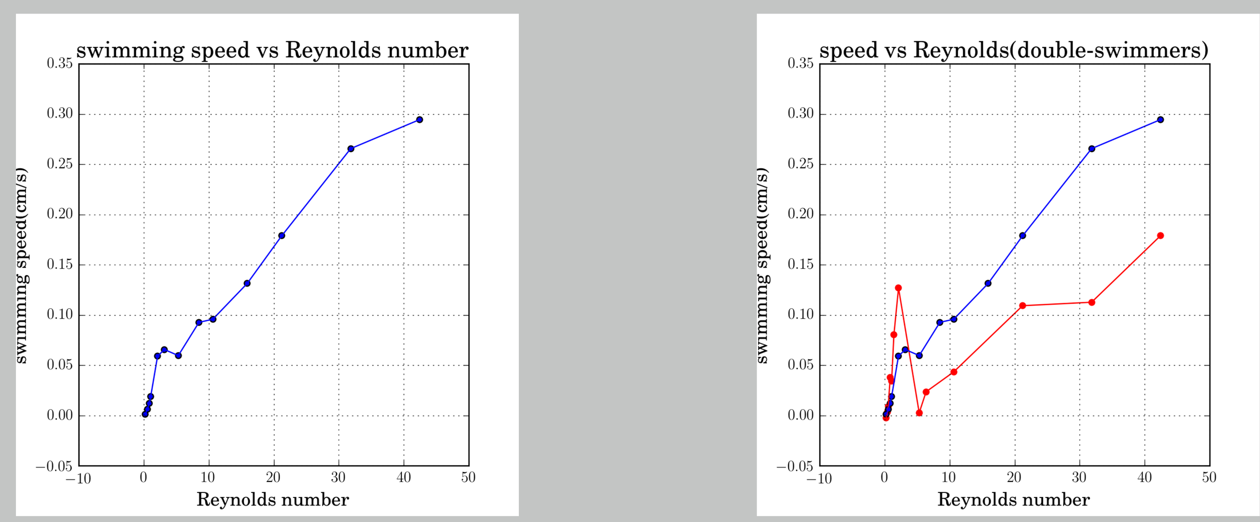

Figure 4: organism speed for different Reynolds numbers, blue: single swimmer, red: double-swimmers

- For small Reynolds numbers swimming speed increases with increasing Re number.
- At **Re** $\approx$ **2** we see a drop in speed, which is the transition from small Reynolds numbers to intermediate.
- Swimming speed improves significantly as a result of swimmers' synchronized motion for small Re.

Simulations: b. suspension

- We investigated the dynamics of suspension of organisms by simulating two suspensions of 10, 32 uniformly distributed with random orientations swimmers in small and intermediate Reynolds regimes.
- For smaller Re it is easier to identify formation of swarms. In higher Re with 10 swimmers there is no swarms identified, whereas with higher concentration of swimmers concentrated groups are easily seen.

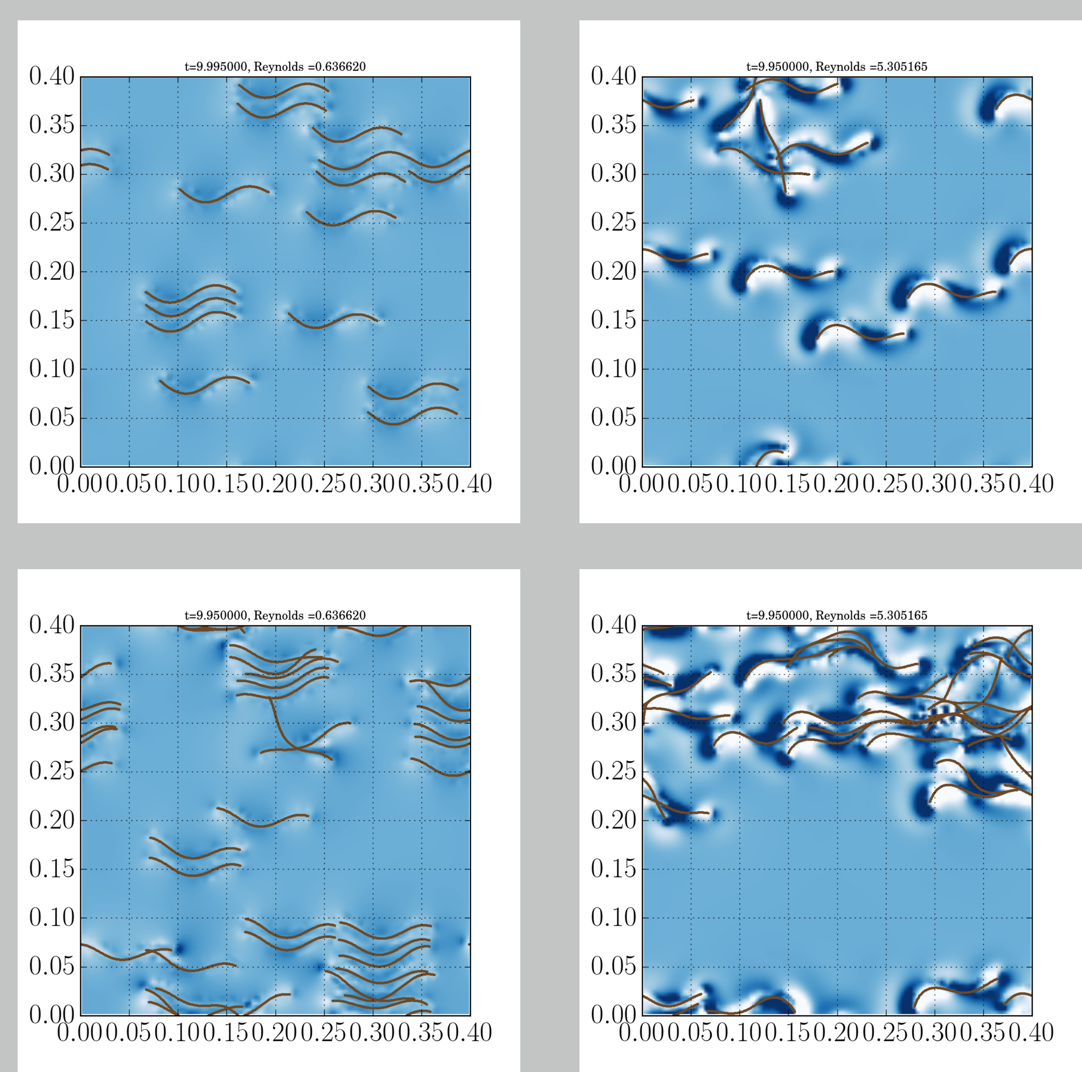

Figure 5: Suspension of organisms, left: 10 , right: 32

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Future Work

- With the method at hand, it is easy to study dynamics in the presence of solid boundaries.
- Study could be extended to investigating 3D simulations using the current software.

References

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