HIRING SCHEDULE OPTIMIZATION AT THE SURREY FIRE DEPARTMENT

BOLONG HE[1,2], SNEZANA MITROVIC-MINIC[1], LEN GARIS[3], PIERRE ROBINSON[4], AND TAMON STEPHEN[1]

ABSTRACT. The Surrey (British Columbia, Canada) Fire Department has an annual cycle for hiring full-time firefighters. This paper optimizes the timing of the annual hiring period. A key issue is handling incapacity absences, which can be covered by overtime of full-time hires.

Short-term and long-term absences patterns are analyzed according to season and age cohorts of the firefighters. These are then used in both an explanatory and time series model to predict future absences. The hiring schedule is optimized based on these predictions and additional constraints.

The current practise fares well in the analysis. For the time period studied, moving to earlier hiring dates appears beneficial. This analysis is robust with respect to various assumptions.

This is a case study where analytic techniques and machine learning are applied to an organizational practise that is not commonly analyzed. In this case, the previous method was not much worse than the optimized solution. The techniques used are quite general and can be applied to various organizational decision problems.

1. INTRODUCTION

The fire department is an indispensable city service. It is also a hazardous industry that has frequent work-related injury rates and is stressful [15]. This means that firefighters have substantial incapacity time due to illness or injury, and also unpredictable absences from work. In 2015, the National Fire Protection Association (NFPA) reported 68,085 firefighter injuries occurred in the line of duty in 2015 in the US [7]. As a result, it is difficult to manage the level of staffing and as well as the personnel budget.

The sponsor of this project, the Surrey (British Columbia, Canada) Fire Department, faces the problem of maintaining an appropriate level of staffing, and calculating an accurate annual budget. They need to fulfil the required level of staffing to satisfy NFPA standards and respond to any emergency immediately that may occur at any time. However, they are also attempting to look for models to help them to predict more accurate staffing levels and calculate a more precise annual budget, and then, find ways to manage the budget.

In this work, we advanced their model in two ways. First, we looked for additional explanatory variables for absences. We found that a helpful variable is the age cohort of the firefighters. This led us to consider the effects of short-term and long-term absences separately, which gives some insight into the pattern of absences. Second, we apply a machine learning approach, time-series forecasting, that learns an absence pattern from historical data to provide a forecast. We then optimize the budget against the forecast to produce a hiring schedule. This provides a modest savings over the previous plan. We examine the solution for sensitivity, and find it to be robust against some natural variations in assumptions. We implemented the forecasting and simulation optimization as a spreadsheet tool.

2. BACKGROUND

2.1. **Previous Work.**

Banjar Management Inc. worked on predicting annual staff operating budgets for Surrey Fire Department in 2002. In their project, they developed a probabilistic model using Monte Carlo simulation to

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[1]DEPARTMENT OF MATHEMATICS, SIMON FRASER UNIVERSITY, 8888 UNIVERSITY DRIVE, BURNABY, BRITISH COLUMBIA V5A 1S6, CANADA

[2]3D MEDICINES CORPORATION, 158 XINJUNHUAN ROAD, UNIT 2A, MINHANG DISTRICT, SHANGHAI, 201114, CHINA

[3]CENTRE FOR PUBLIC SAFETY AND CRIMINAL JUSTICE RESEARCH, UNIVERSITY OF THE FRASER VALLEY, ABBOTSFORD, BRITISH COLUMBIA V2S 7M8, CANADA

[4]SURREY FIRE DEPARTMENT, 8767 132 STREET, SURREY, BRITISH COLUMBIA V3W 4P1, CANADA

E-mail addresses: `ihebolong@gmail.com`, `snezanam@sfu.ca`, `lwgaris@surrey.ca`, `PBRobinson@surrey.ca`, `tamon@sfu.ca`.

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predict firefighters absences due to sickness and work-related injuries based on the monthly average sick time in 1998-2001. Then, they used this projected data to estimate staffing pools cost. They predicted the level of absence due to sickness and work-related injuries, using the factors of seasonality (i.e. the absences differ from month to month) and shifts (i.e. the absences differ between shifts). We believe that seasonality and shifts are not enough to describe the fluctuation of the absences, and other as yet unidentified factors may also be relevant.

On the theoretical side, Fry, Magazine and Rao [6] produced a model for determining the optimal staffing level at fire department, and applied it to the operations of the Cincinnati Fire Department. As Banjar Management did, they build a stochastic model for firefighter absences and use this to forecast costs and set the hiring level. They treat both absences and workforce attrition (including retirements) as random events. They work with a fixed hiring cycle, and thus do not incorporate seasonal effects into their analysis.

More generally, this staffing problem can be understood in the paradigm of inventory theory, see for example [13], and in particular, the *newsvendor* model.

2.2. **Project Context.** In the personnel budget of the Surrey Fire Department, beside regular salary, there are two other parts: the staff pool costs for the absences due to illness or injury, and the costs to cover other absences such as vacation, family leaves, etc. Between these two absence costs, the former contains more uncertainty. Finding factors to explain these uncertainties, and then predicting the staffing level is the first goal of this research project.

The Surrey Fire Department has around 360 firefighters in total. Over time, people leave the force for retirement or other reasons. Instead of hiring new employees immediately when they have vacancies, they currently recruit new members once a year. Before the annual recruitment, they will sometimes face a deficit of personnel and use overtime to deal with this problem. In some cases, they realize that the overtime cost for filling these vacancies is less than the cost of hiring a new firefighter. Additionally, too much overtime will increase the stress of the firefighters. A goal of this research is to make a hiring schedule based on the predictions for the staffing pool, the aim is to keep an appropriate level of staffing to deal with the vacancies and unpredictable employees injuries, and also to lower the annual budget.

The Surrey Fire Department provided their historical data from January 2014 to December 2016 at the beginning of this project. The historical data includes anonymized personnel records and their budget plan. Our analysis is based on these 3-year data. Towards the end of the project, they provided their 2017 data from January to September. We use 2017 data to validate our model.

3. ANALYSIS OF FIREFIGHTER INCAPACITY ABSENCES

The Surrey Fire Department is organized around half-day shifts. Each firefighter works for 4 days and then they have 4 days off. The firefighters will alternate between day and night shifts over a 16 day period to complete one cycle. In this project, we define *incapacity absences* as missing a scheduled shift due to illness and injuries.

When the Fire Department has a staffing shortage due to incapacity absences, they will ask an off-duty firefighter to work overtime, sometimes referred to as call-back duty. This is where the extra operating cost is incurred.

To understand the fluctuation of the incapacity absences due to illness and injuries, we will analyze the absences from 2014 to 2016 and their dependency on the following dimensions: shifts, months, and years of birth. In addition, we will also analyze the long-term absences separately.

3.1. **Overview.** The Surrey Fire Department provided their three previous years of data to us. This data is from January 2014 to December 2016. In total, there 403 firefighters worked during these three years. The birth years of these firefighters are in the range of 1954 - 1995. These 403 firefighters took 76476.75 hours of incapacity absence in these 3 years. The numbers of incapacity absence hours for each single year are shown in Table 1.

Year	Incapacity Absence Hours	Number of Firefighters Working	Average Hours Per Firefighter	% Change in Avg. Hours from Previous Year
2014	22,428	366	61.3	-
2015	27,157	374	72.6	18.5%
2016	26,892	372	72.3	-0.4%

TABLE 1. Summary of the Absence Hours from January, 2014 to December, 2016

If we count the number of months that each firefighter had at least one incapacity absence during 2014 - 2016, 170 firefighters (42.2%) had 0 incapacity absences; 306 of them (75.9%) had incapacity absences in less than 3 months; 45 of them (11.2%) had incapacity absences in more than 7 months; and 10 (2.5%) of them had incapacity absences in more than 12 months. This is summarized in the histogram in Figure 1.

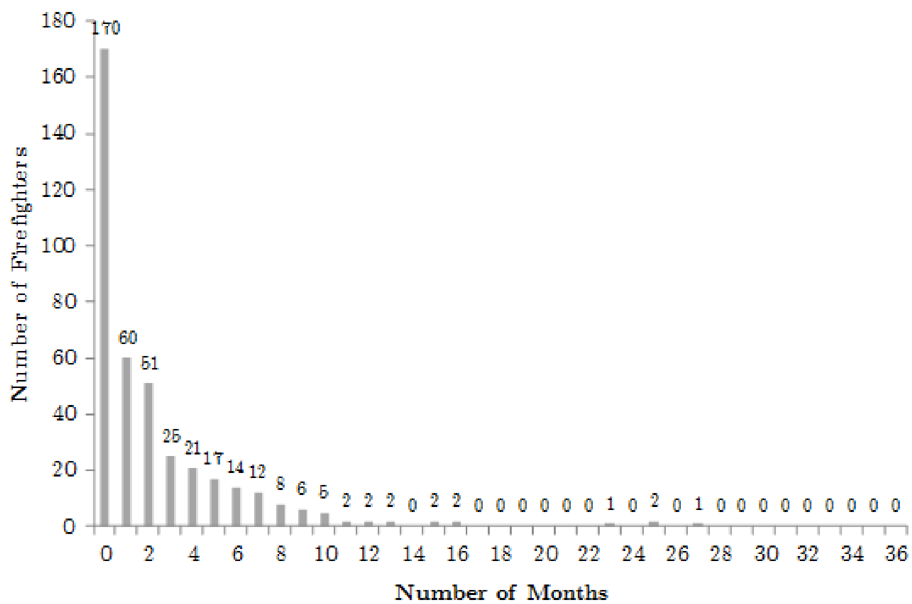

FIGURE 1. Number of Firefighters with Absences in the Given Number of Months

Counting by total number of incapacity absence hours 2014-16, 170 of the firefighters (42.2%) had zero incapacity absence hours; 329 firefighters (81.6%) had less than 240 incapacity absence hours (i.e. 20 shifts); and 36 firefighters (8.2%) had more than 600 incapacity absence hours (i.e. 50 shifts). The histogram is shown in Figure 2.

From our analysis, 67.3% of the total incapacity absence hours are taken by 10% of the firefighters. We believe that these firefighters got either serious injuries or illnesses that need a long time to recover from. In Section 3.4, we will evaluate these long-term absences separately.

3.2. **Seasonality of Incapacity Absence.** Season is one factor believed to influence the firefighter incapacity time. First, some types of fires are seasonal, for instance wildfires and fires due to outdoor activities. When the number of fires increase, there will be more chance for a firefighter to get sick or injured. Second, some illnesses are also seasonal. For instance, flu may increase the incapacity absence hours in specific period of a year.

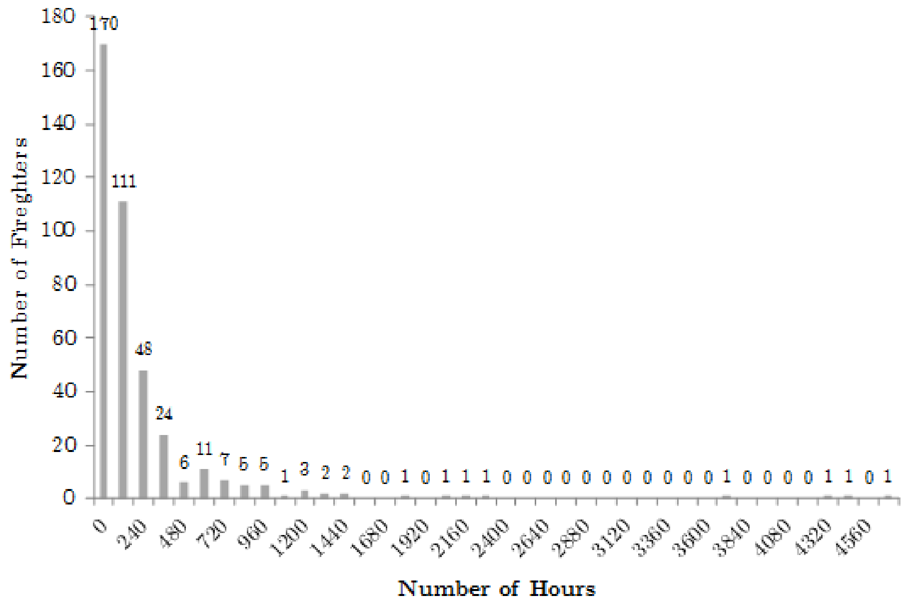

FIGURE 2. Number of Firefighters with Incapacity Absence Hours in the Given Range

In this section, we will analyze the hours of sickness absences monthly from January 2014 to December 2016. We will use a calendar year as a cycle to analyze our result.

3.2.1. *Methodology.* To calculate the total incapacity absence hours for each month, we do the following steps:

a) Identify the active firefighters from the data, and sum up the hours of incapacity absences by month for each individual firefighter.
b) For each single month, we exclude the firefighters retired and newly hired during that month, because these firefighters will affect the average hours.
c) We sum up the absence hours for all firefighters for each month, and divide by the number of firefighters working in the corresponding month and the number of days in that month.

3.2.2. *Results and Analysis.* We firstly inspect all the firefighters in the Fire Department, see Figure 3. We include the average values for the three years (the dotted line in the charts). The fluctuation of the incapacity absence hours is significant in our result. However, the number of absence hours tends to decline for the first half of every year, and approach lowest around July; then the number of absence hours tends to rise in the second half of every year, and approaches its highest point around January.

Next, we analyze the people in the four shift cohort separately. We use the same method but apply it to individual shifts. This showed differences between the shifts, but not consistent or pronounced. Ultimately, shift cohort was not helpful as an explanatory variable. We have omitted the details of this analysis in the interest of brevity.

3.3. **Age and Cohort Effect on Incapacity Absences.** Age is another important factor that influences the incapacity time. As the age of a firefighter increases, physical fitness may decline, and the likelihood of illness or injuries may increase. In this section, we study the hours of incapacity absences by the birth years of the firefighters.

3.3.1. *Methodology.* We only include the firefighters born between 1957 and 1991 for this analysis. We perform the following steps to calculate the hours:

a) Identify the firefighters working from January 2014 to December 2016.
b) For each month, exclude the firefighters that are newly hired or that left the force within that month. In other words, for each firefighter, we exclude the months he/she did not work in the Fire Department for the full month.

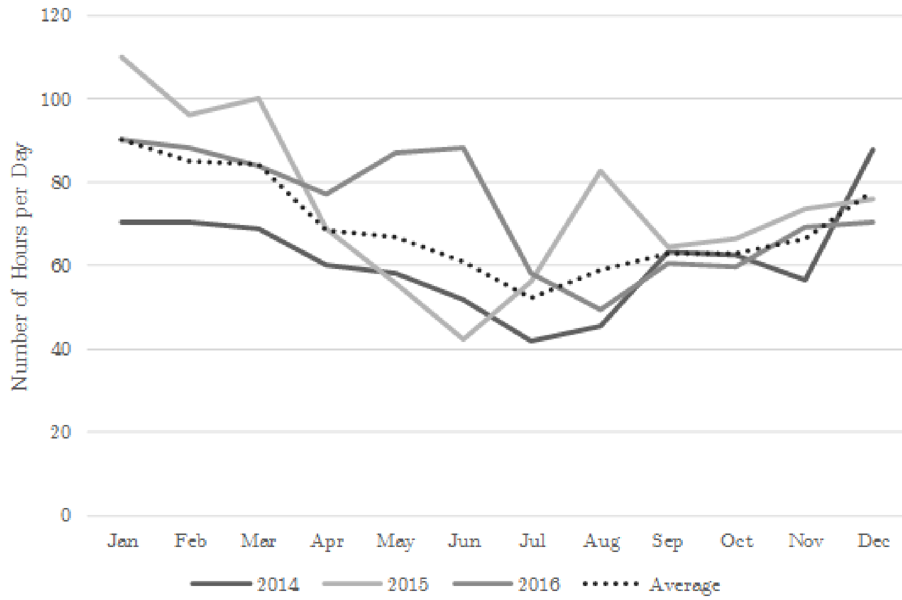

FIGURE 3. Average Incapacity Hours per Day by Month

c) Instead of evaluating the firefighters in each single birth year, we group the firefighters into 5-year ranges, and get 7 groups.
d) Sum up the total incapacity absence hours for each group and divide by the number of days that the firefighters worked in the corresponding group.

3.3.2. *Result and Analysis.* Figure 4 summarizes our results. Except the groups of 1957-1961 and 1972-1976 the incapacity absence hours are low and distributed fairly uniformly. For the group of 1957-1961, it matches our expectation that as the age of a firefighter increases, physical fitness declines, and the likelihood of illness or injury increases. For the group of 1972-1976, the higher incapacity absence hours are due to more long-term absence hours. We believe that the reason is they are towards the end of the period of their career where they undertake the most physical firefighting tasks. Older firefighters are promoted to a more supervisory role.

3.3.3. *Average Age Compared with Total Absence Hours.* We additionally ran a comparison of average age for each month with total absence hours. We observed that the average age drops when the Fire Department hires new firefighters in June or July, and that the average absence hours drop around the time the average age drops. This is consistent with what is observed in the cohort-based age analysis. Currently, the Fire Department hires new firefighters during the summer, this hiring cycle is likely another source of seasonal variation in the incapacity absence rates.

3.4. **Firefighters with Long-term Absences.** In Section 3.1, we mentioned that 10% of the firefighters took 67.3% of total incapacity absence hours from January 2014 to December 2016, and we believe that the reasons for these absences are serious injuries or illness.

We identify the absences that take more than 16 shifts off consecutively as *long-term incapacity absences.* In other words, these absences are longer than 1 month. According to our definition, 67.5% of incapacity absence hours from January 2014 to December 2016 are long-term absences. In this section, our analysis focuses on these long-term absences.

If we exclude the long-term absences and evaluate the seasonality of the remaining absence hours, the results are shown in Figure 5. We use the same vertical axis as in Figure 3 for better comparison. After the long-term absences are removed, the monthly incapacity absence hours changes from the range 60-80 to around 20, and the fluctuation is reduced significantly. For 2015 and 2016, the trend is similar to the previous graph. For 2014 the trend is different where the average hours are higher in the first half of the year.

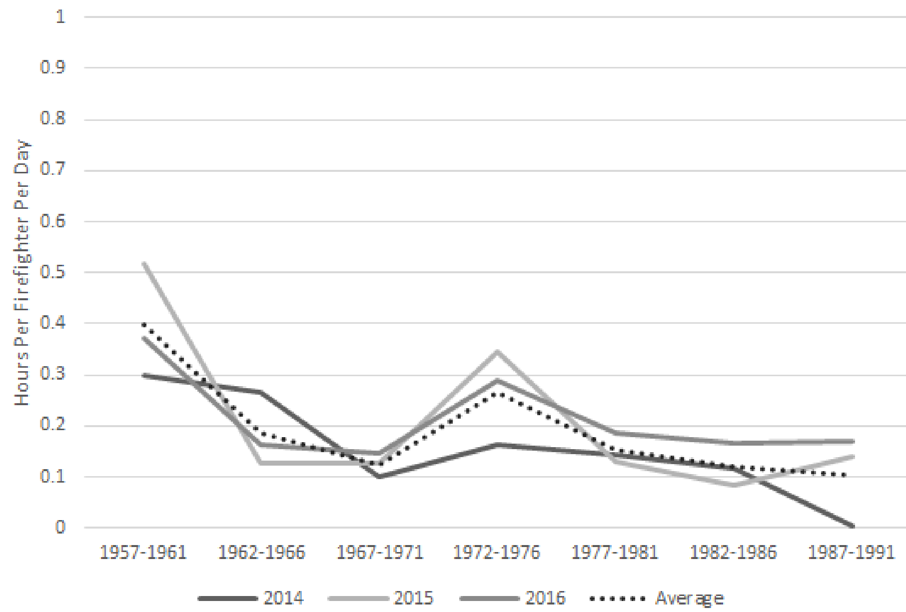

FIGURE 4. Average Incapacity Hours per Day by Month by Age Cohort

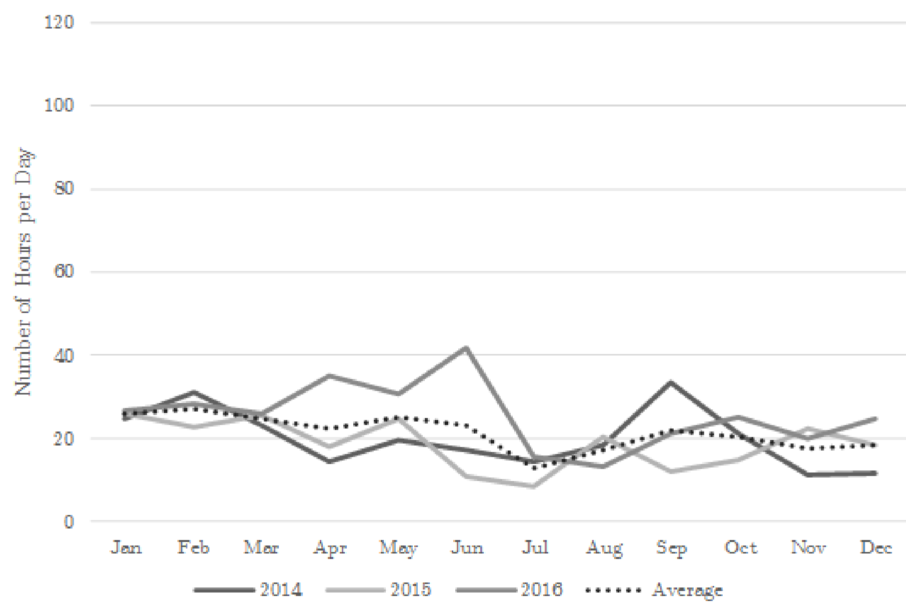

FIGURE 5. Average Incapacity Hours per Day without Long-term Incapacity by Month for all Shifts

Considering only the long-term absences, details are shown in Figure 6 and Figure 7. Figure 6 shows the number of firefighters taking long-term incapacity absence for each month and Figure 7 shows the number of hours per day for each month. These two graphs are similar because most of the long-term absences took the whole month off. In these two graphs, although the average line shows the absence hours during the summer time to be lower, the lines for each of the 3 years (2014 - 2016) are quiet different. We believe that these absences are due to serious injuries or illnesses, and these injuries or illnesses happen fairly evenly through out the year.

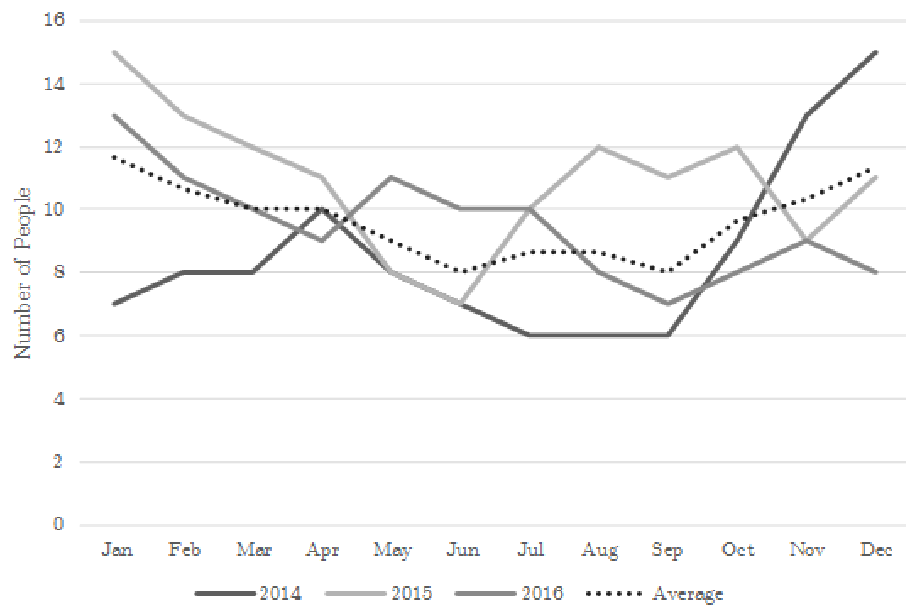

FIGURE 6. Numbers of firefighters Taking Long-term Incapacity Absence for Each Month

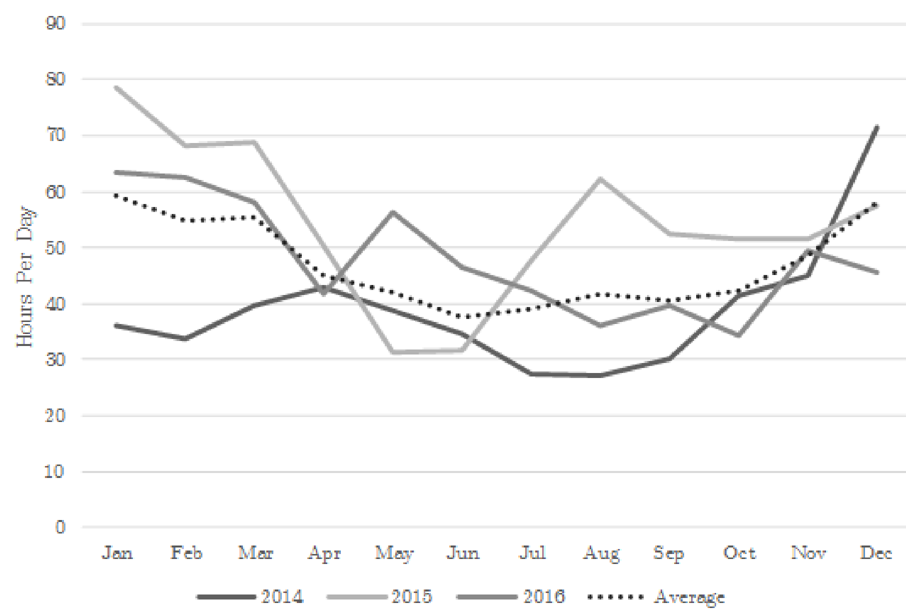

FIGURE 7. Numbers of Long-term Incapacity Absence Hours per Day for Each Month

In Figures 5 and 7, the average line shows a similar trend to the one observed in Section 3.2. The absence hours decrease in the first half of a year, and increase in the second half of a year. We only have 3 years of data, but for each graph, 2 of these 3 years will follow this pattern. We believe that there exists a seasonal behaviour in both of the short-term and long-term absence hours, although it is not very strong.

Next we look at the age and cohort effect in the long-term absences, the result is shown in Figure 8. From the average line, the incapacity absence hours tend to be uniform but still increasing a little bit as the age of the firefighters increases. For the increase in the youngest group, we believe it is because the sample size for that group is small.

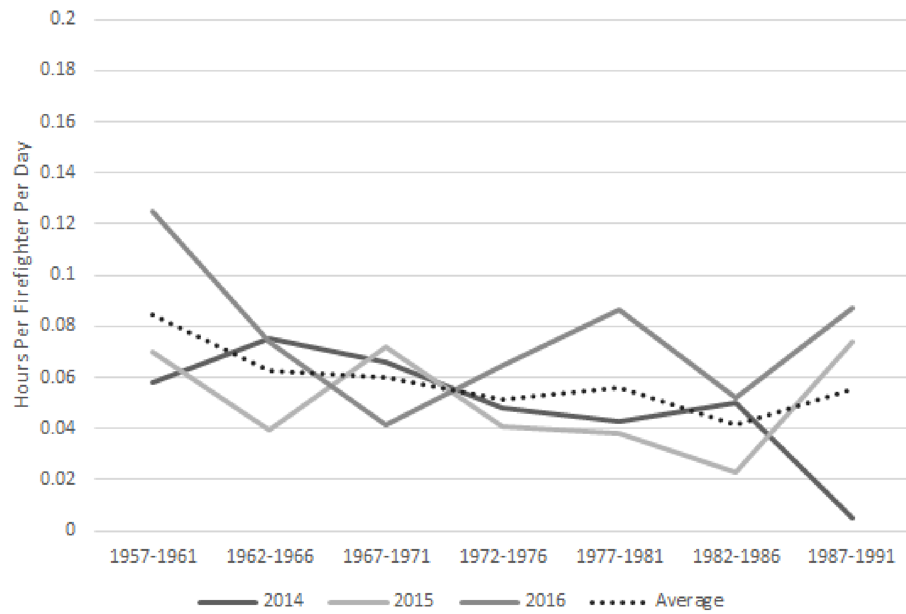

FIGURE 8. Average Incapacity Hours per Day by Age Cohort for Short-term Absences for all Shifts

4. FORECASTING INCAPACITY ABSENCES

In this section, we forecast the incapacity absence hours in the year 2017 from the 2014 - 2016 data. We also validate against available 2017 partial data and assess how effective such a forecast will be for future years. We then provide a model for forecasting future years.

4.1. **Forecasting Method Selection.** Forecasting techniques may be divided into two major categories: quantitative forecasting and qualitative forecasting [10]. Quantitative forecasting methods are applied when the historical numerical information is available and one can expect that the past pattern will continue in the future. In contrast, qualitative forecasting methods are based on the opinion and judgement of experts. These method are usually applied when there is little or no quantitative information available. Qualitative forecasting can also be applied to adjust a quantitative forecast when some information is not suitable for inclusion in the quantitative forecasting model. In this project, since we have good historical data, and we believe that the patterns will continue in the future, we use quantitative forecasting methods.

There are two major classes of quantitative forecasting methods: explanatory models and time series [10]. Explanatory models assume that the variable to be forecast has an explanatory relationship with particular independent variables. In an explanatory model, the chosen variables will rarely capture the entire picture: some aspects will remain unaccounted. These are included in an error term.

The previous Monte Carlo simulation for the Surrey Fire Department is an explanatory model. The model used historical average monthly sickness absence hours, separated by shift, and it assumes that the absence hours for the following year are related to the previous average. To improve this model inside the simulation paradigm, we could find more variables that may affect the sickness absence hours and further divide the data along those lines. However, the previous model is already slicing the data thinly. Also, we have observed that two-thirds of the absence hours are long-term absences. We believe that the reason for these absences are unexpected injuries or illnesses, and there are few obvious patterns in them.

Compared with explanatory models, a time series forecast treats the system as a black box, using only historical information on the variable to be forecast, and does not attempt to determine the factors affecting its behaviour [10]. The objective of time series forecasting methods is to discover the pattern in the historical data series using machine learning and extrapolate that pattern into the future. Time series are useful when the causes of changes are not well understood, or it is difficult to describe the relationship assumed to govern its behaviour. Often, the main goal is to forecast rather than to find the underlying causes. Time series forecasting models have been given various names in different disciplines [8]. They are known as dynamic regression models, panel data models, longitudinal models and transfer function models.

Since there may be hidden patterns in the data in this project, we decided to use time series forecasting. From the analysis in Section 3, we observed a seasonal effect in our historical data, and we use the SARIMA (Seasonal Auto Regressive Integrated Moving Average) model to forecast the future absences.

4.2. **Time Series Forecasting Model.** Time series forecasting uses historical information on the variable to be forecast, and does not identify the explicit factors which affect its behaviour. The model extrapolates recurring trends of a time series in order to infer its future behaviour, controlling the influence of one-time bumps. Time series forecasting models have been widely used in business and industry [11], for example, forecasting air transport demand [12], and decision-making of a corporate entity [9]. Time series models we considered for forecasting include ARIMA models, and SARIMA models.

4.2.1. *SARIMA Model.* The Seasonal AutoRegressive Integrated Moving Average (SARIMA) model forecasts the time series by fitting the past data to the model. A SARIMA model can be broken down into four parts: the AR model, the MA model, the integrated model, and the seasonal model.

An AutoRegressive model (AR) is a stochastic model which is based on time-lagged forecasts of the series. Box and Jenkins [1] give a generalized form of an AR model of order p or AR(p) model,

$$x_t = c + \varphi_1 x_{t-1} + \varphi_2 x_{t-2} + \ldots + \varphi_p x_{t-p} + \varepsilon_t \tag{4.1}$$

where c is a constant, and ε_t is an error term. In an AR(p) model, we use p values of the time series before x_t, that is, $x_{t-1}, \ldots, x_{t-p}$ as predictors, called *time-lagged parameters.* The AR coefficients $\varphi_1, \ldots, \varphi_p$ give time series patterns. The *order p* in AR(p) model indicates the order of the autoregressive process, i.e. the number of previous terms of the process takes into account. For example, an AR(1) model has the form $x_t = c + \varphi_1 x_{t-1} + \varepsilon_t$. In this model, the variable x_{t-1} in the previous time period of x_t has become the predictor.

The inputs for an AR(p) model are the time series and the order p. The model regresses the time series to solve the remaining variables: c, ε_t, and the AR coefficients $\varphi_1, \ldots, \varphi_p$.

In an AR model, the parameter p is chosen based on the graph of partial autocorrelation function (PACF) of the time series. In this project, in order to simplify the process for future users, we write a Python function to fit the model automatically using an information criterion. See Section 4.2.3 for details.

A Moving Average (MA) model is another type of stochastic time series model which regresses against the past errors of the time series. Different from an AR model which use past values in a regression, an MA model uses past forecast errors in a regression-like model. Its generalized form of order q is given by:

$$x_t = c + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \ldots + \theta_q \varepsilon_{t-q} \tag{4.2}$$

where $\varepsilon_{t-1}, \ldots, \varepsilon_{t-q}$ are white noise, and $\theta_1, \ldots, \theta_q$ are the MA coefficients of the model. In other words, this process is given by a weighted average of the noises. The order q in MA model refers to the highest order power in the polynomial.

Similar to the AR model, the inputs for an MA(q) model are the time series to be analyzed and an order q. The model regresses the time series to solve the remaining variables: c, and the MA coefficients $\varepsilon_{t-1}, \ldots, \varepsilon_{t-q}$.

The parameter q in MA model, is chosen based on the graph of autocorrelation function (ACF) of the time series. In this project, in order to simplify the process for future users, we write a Python function to fit the model automatically using information criterion. See Section 4.2.3 for details.

If we combine AR(p) model and MA(q) model together, they produce an ARMA model with an order (p, q), and can be written as,

$$x_t = c + \varphi_1 x_{t-1} + \varphi_2 x_{t-2} + \ldots + \varphi_p x_{t-p} + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \ldots + \theta_q \varepsilon_{t-q} + \varepsilon_t \tag{4.3}$$

However, there is a limitation in ARMA models, which is that the ARMA model requires the time series to be *stationary.* In order words, the mean and the variance of time series used in ARMA models do not change as the time series evolves over time. When the time series is not stationary, as is the case in our time series an improved model is needed. A model proposed by Box and Jenkins [1], called AutoRegressive Integrated Moving Average

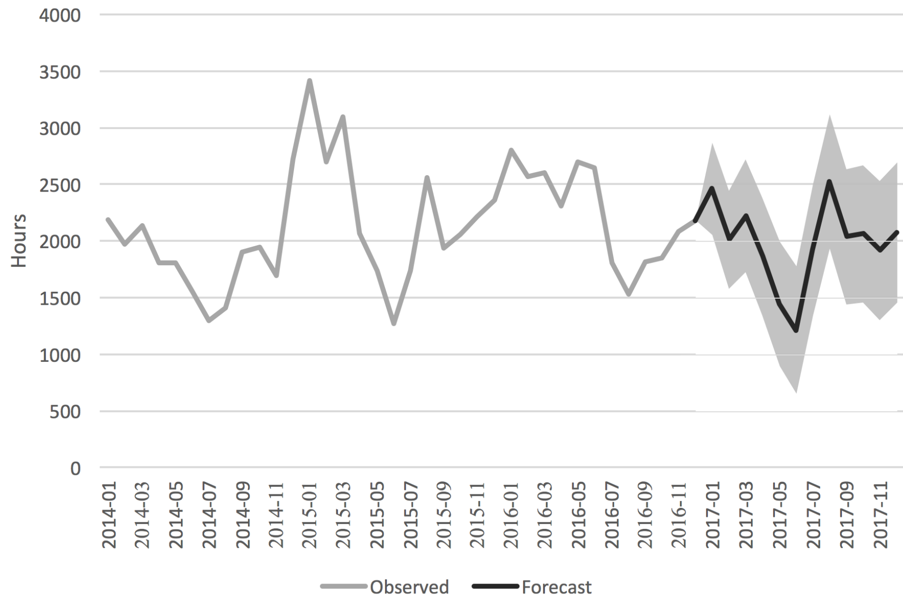

FIGURE 9. Forecasting Result of SARIMA$(2,1,1)x(2,0,0,12)$

(ARIMA) model, outperforms the ARMA models because it can deal with non-stationary time series. It works by differencing the series until stationarity is achieved [1].

From the analysis in Section 3, we believe a seasonal behaviour exist in our time series. In this case, an extension of the ARIMA model, Seasonal AutoRegressive Integrated Moving Average (SARIMA) model would be the most appropriate to use. The model have parameters p, d, q, P, D, Q as well as a seasonality parameter s, It can be denoted as SARIMA$(p, d, q) \times (P, D, Q, s)$. The seasonal part in the model may be identical with the non-seasonal part and may as well include a seasonal autoregressive term, a seasonal moving average term, and a seasonal difference operator. To distinguish them, the seasonal parts are denoted with capital letters P, D, Q representing the seasonal AR, the seasonal difference operator, and the seasonal MA term, respectively. The seasonality parameter s is the number of periods per season, in our case $s = 12$.

4.2.2. *Identifying the Parameters in SARIMA model.* In this project, in order to simplify the process for future users, we use a modified version of the code of Daxiongmaotaosha [4]. This code fits the model automatically using *Akaike's Information Criterion (AIC)* , see for example [3]. In SARIMA$(p, d, q) \times (P, D, Q, s)$ model, we have seven parameters. We already know the seasonality parameter $s = 12$. We use an Augmented Dickey-Fuller test [5] to verify that first order differencing is enough to stationarize the time series, and we set $d = 1$ in our SARIMA model.

Then, we try the combinations of p, q as well as the seasonal part P, D, Q, and choose the model with smallest AIC. The larger number we choose for these parameters, the smaller AIC we may get. However, to avoid overfitting, especially, when the the historical data set is small, in practice, the parameters are normally restricted to small values.

4.2.3. *Forecasting and Validating the SARIMA Model.* We first forecast the 2017 absence hours based on the 3-year historical data. After a few experiments on the SARIMA model, using a customized version of Brownlee's [2] code, we noticed that when the degree of seasonal difference is greater or equal than 2, it leads to an unexpected results. Thus, we restrict D to 0 or 1, and set $p, q, P, Q \in \{0, 1, 2, 3, 4\}$. Some results of our experiments are shown in Figure 9 and 10. The grey area is the 10% confidence interval.

It is good practice in time series forecasting to cut off the last period in the time series to benchmark the forecast. However, we only have three periods (years) of data. If the last period of the data is cut off, we only have two periods, and this is too small for the SARIMA model to discover the pattern in the data [14].

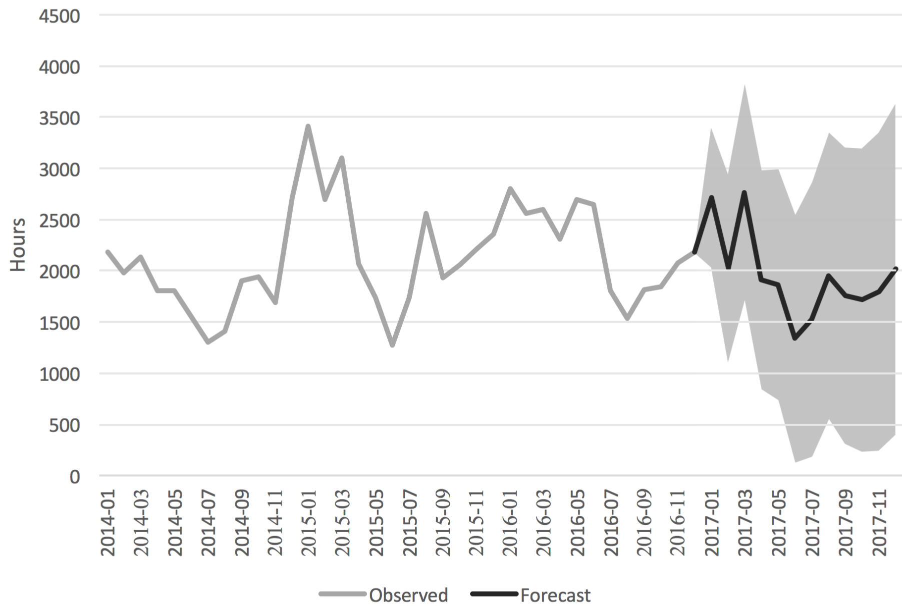

FIGURE 10. Forecasting Result of SARIMA$(4,1,0)\times(1,1,0,12)$

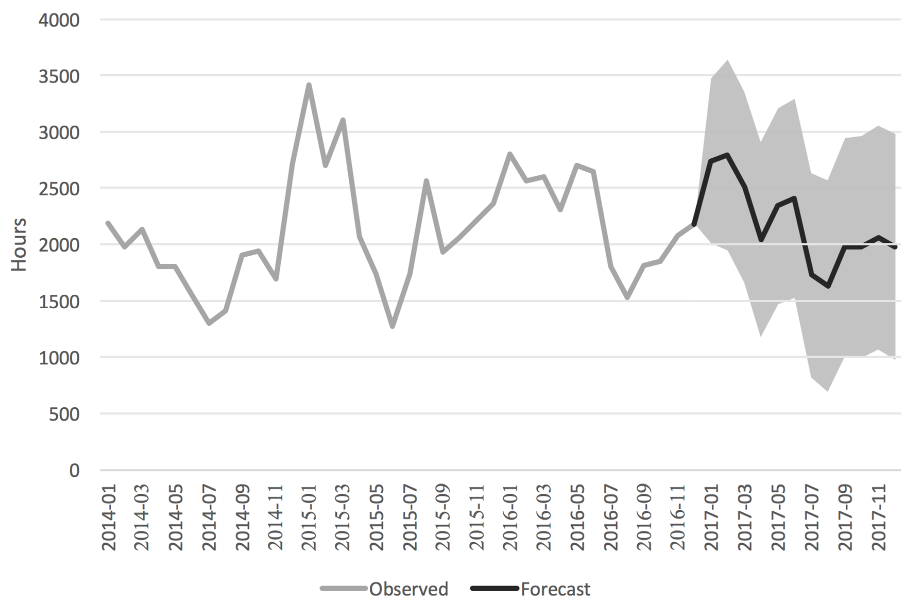

FIGURE 11. Forecasting Result of SARIMA$(3,1,4)\times(0,1,0,12)$

The Surrey Fire Department provided their 2017 data from January to September as we were finishing this report. We used this updated data to improve our auto-fitting function by evaluating the root-mean-square error between the forecast results and the actual data. After our experiments, again using a version of Brownlee's [2] code, the optimal parameters for the model are estimated to be SARIMA$(3,1,4)\times(0,1,0,12)$ while we restrict D to 0 or 1, and set p,q,P,Q up to 6. This is the forecast shown in Figure 11.

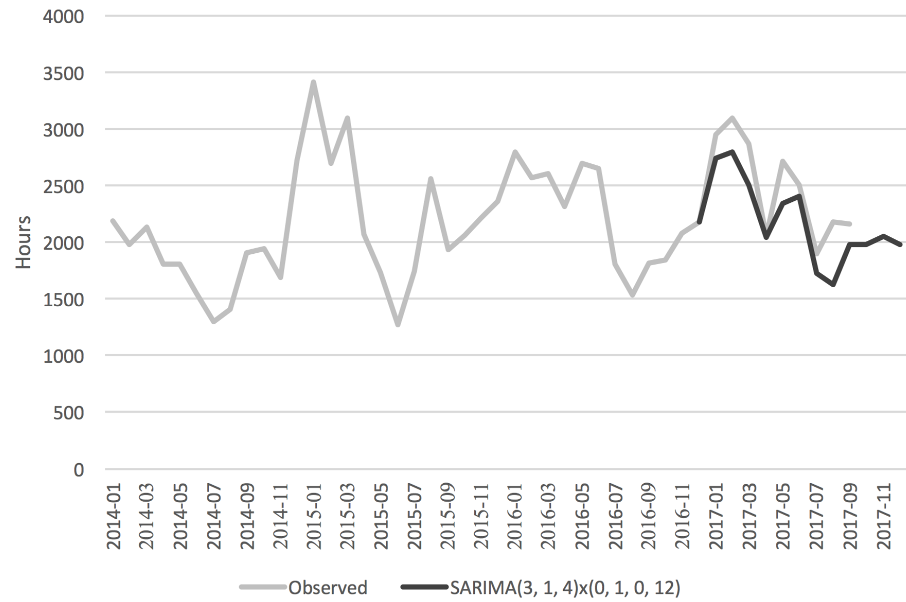

FIGURE 12. Comparison between the actual data and SARIMA$(3,1,4)\times(0,1,0,12)$

Comparing the actual data with the forecast results of SARIMA$(3,1,4)\times(0,1,0,12)$ that are shown in Figure 12, the trends are similar, but the results in SARIMA$(3,1,4)\times(0,1,0,12)$ model are slightly lower.

5. Optimizing the Hiring Schedule

In this section, we optimize the hiring schedule using our forecast result based on a SARIMA$(3,1,4)\times(0,1,0,12)$ model.

5.1. **Personnel Budget Calculation.** The Surrey Fire Department calculates their annual staff operating budget by month, and divides the budget into four parts: anticipated regular salary, operational gapping saving, staffing pool cost, and overtime coverage cost.

The *anticipated regular salary* S^r_m for month m is based on the salary and benefits for each firefighters and the number of firefighter positions in the fire department. Depending on the length of service and positions, the Surrey Fire Department divides their firefighters in to eight salary groups. These eight groups of firefighters have different level of salaries and benefits. The Fire Department calculates this anticipated regular salary based on the distribution of the these groups, using the monthly salary for 4th year and 10th year service length groups. We denote the monthly salary for these two groups as s^4 and s^{10}.

When the Surrey Fire Department has firefighters leave the force for retirement or other reason, before they hire new firefighter to fill these positions, they will have the *operational gapping saving* for the gapped positions. Once we know the retirement date of each firefighter in the fire department we can calculate the number of firefighters that retire in each month and the number of operational gaps. We use the operational gappings multiplied with the 15-year firefighter salary to calculate the operational gap saving.

Denote the number of gapped positions in month m as $n^g_m, m=0,1,\ldots,36$; the number of firefighters leaving the force in month m as $n^r_m, m=1,2,\ldots,36$, and the number of newly hired firefighters in month m is $n^h_m, m=1,2,\ldots,36$. Then the number of gapped positions in this month is:

$$n^g_m = n^g_{m-1} + n^r_m - n^h_m \tag{5.1}$$

and the operational gapping saving S^g_m for month m is:

$$S^g_m = n^g_m \cdot s^{15} \tag{5.2}$$

Based on the vacation, training, and statutory holiday hours, the incapacity absence hours, the number of gapped positions, and the total firefighter positions they will calculate the number of firefighters can be on duty in each working shift, we denote this by n_m^o for month m. Then, compare this number with the number of firefighters that need to be on shift n_m^r for month m, if vacancies exist, they will use overtime to fill the vacancies, and the vacancies denote by:

$$n_m^v = n_m^r - n_m^o \tag{5.3}$$

This overtime cost is called the *staffing pool cost* S^p. The Fire Department uses the salary s^{10} to pay these overtimes. Since there are 182.7 shifts in one year, the average salary for one shift is:

$$s^{10_{avg}} = s^{10} \cdot 12/182.7 \tag{5.4}$$

The staffing pool cost for month m is calculated by multiplying the number of vacancies by the average salary for each shift and the number of shifts n_m^s in month m, i.e.:

$$S_m^p = n_m^v \cdot s^{10_{avg}} \cdot n_m^s \tag{5.5}$$

Lastly, the *overtime coverage cost* S_m^o for month m is the cost that covers the personal leaves of each firefighter, including family leaves, bereavements, etc. This is similar to the staffing pool cost, but when they have vacancies to fill, they use s^4 to pay the overtime.

$$S_m^o = n_m^p \cdot s^{4_{avg}} \cdot n_m^s \tag{5.6}$$

where n_m^p is the vacancies for personal leaves, $s^{4_{avg}}$ is the average salary for one shift based on s^4, and n_m^s is the number of shifts in month m.

The total cost for month m is:

$$S_m = S_m^r - S_m^g + S_m^p + S_m^o \tag{5.7}$$

5.2. **Optimization Model and Result.** According to the Surrey Fire Department, the vacation, training, statutory holiday portion of the staffing pool cost calculation as well as the family leaves, and bereavements in the overtime coverage cost can be forecast and to some degree managed through scheduling. It is more difficult for them to understand and forecast the incapacity absence hours, which is a major factor in planning the hiring cycle. We will use the forecast result from Section 4 for the incapacity absence hours, to propose a hiring schedule to optimize the budget.

To minimize their training cost, the Surrey Fire Department will only hire new firefighters once a year. In addition, they prefer to hire at least 8 firefighters each time due to high training cost. Since there are around 16 firefighters that leave the force every year, we evaluate the expected budget for all hiring schedule possibilities: a single annual hiring month from January to December, and the number of hires from 8 to 20. We also require that the number of vacancies each month not exceed 20, to prevent overextension of firefighters through heavy overtime. Afterwards, we choose hiring schedule with minimal cost satisfying the requirements.

We implemented our algorithms in Python 2.7. A grid showing results for each feasible input, along with the optimal schedule obtained, is shown in Figure 13.

In the results, the light grey cells are the solutions where all vacancies are filled when recruit new firefighters, and the dark grey cell is the optimal schedule that suggest to recruit 11 firefighters in May. Compared with the current plan to hire 16 firefighters in June, our hiring schedule saves $172,461, which is 0.38% of the total budget. Although it is not a large percentage, it seems reasonable. It also shows the current hiring schedule of the Surrey Fire Department performs well but not optimally.

We also examine if the hiring schedule of 2018 will affect the hiring schedule of 2017. We optimized the hiring schedule which gives the least the total budget for 2017 and 2018 with the restriction that the fire department will only recruit once a year and vacancies never exceed 20. The result is shown in Figure 14.

The optimal hiring schedule is to hire 9 firefighters in January, 2017, and to hire 9 firefighters in March, 2018. In this case, the hiring schedule in 2018 will affect the optimal hiring schedule in 2017. This suggests that when the fire department plans their hiring schedule, they should consider the effects of the second year, and of subsequent years.

Number of Hiring	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
2017/01	44720838.50	44726241.41	44812471.78	44737047.24	44823277.61	44909507.97	44995738.34	44920313.80	45046957.90	45092774.54	45179004.90	45143994.09	45270638.19	45397282.28	45483512.65	45529329.29	45655973.39	45742203.75
2017/02	44717236.56	44752499.52	44828176.22	44742198.00	44817874.69	44893551.39	44969228.08	44883249.86	44999340.28	45034603.25	45110279.94	45064715.46	45180805.88	45296896.30	45372572.99	45407835.96	45523926.38	45640016.80
2017/03	44713634.61	44778757.63	44843880.65	44747348.76	44812471.78	44837181.07	44902304.09	44805772.20	44911308.95	44976431.96	45041554.98	44985436.82	45090973.57	45196510.31	45261633.33	45286342.62	45391879.37	45497416.11
2017/04	44710032.67	44764602.02	44819171.36	44712085.80	44807068.87	44861638.21	44916207.55	44768708.26	44863691.33	44918260.68	44972830.02	44906158.18	45001141.26	45096124.33	45150693.67	45124435.56	45219418.63	45314401.70
2017/05	44706430.73	44750446.40	44794462.07	44676822.83	44761252.23	44845681.62	44889697.29	44772058.05	44856487.45	44900503.12	44944518.79	44867293.28	44951722.67	45036152.07	45080167.74	45083769.68	45168199.08	45212214.75
2017/06	44702828.79	44776704.51	44850580.23	44681973.59	44755849.31	44829725.03	44903600.76	44775407.84	44849283.57	44923159.29	44997035.01	44868842.10	44942717.82	45016593.54	45090469.26	45083517.53	45157393.25	45231268.97
2017/07	44699226.85	44762548.89	44825870.94	44727538.08	44790860.13	44813768.45	44877090.49	44778757.63	44842079.68	44905401.73	44968723.78	44910804.64	44974126.69	44997035.01	45060357.06	45083265.38	45146587.42	45209909.47
2017/08	44695624.90	44748393.28	44801161.65	44732688.84	44785457.21	44797811.86	44850580.23	44782107.42	44834875.80	44887644.17	44940412.54	44912353.46	44965121.83	44977476.48	45030244.85	45083013.23	45135781.60	45188549.97
2017/09	44692022.96	44734237.66	44776452.36	44737839.60	44780054.30	44781855.27	44824069.97	44785457.21	44827671.91	44869886.61	44912101.31	44913902.28	44956116.98	44957917.95	45000132.65	45042347.35	45084562.04	45126776.74
2017/10	44688421.02	44720082.04	44751743.07	44742990.36	44774651.39	44765898.68	44797559.71	44788807.00	44820468.03	44852129.05	44883790.07	44875037.37	44906698.40	44897945.69	44929606.72	44961267.74	44992928.76	45024589.79
2017/11	44644405.35	44665512.70	44686620.05	44667313.67	44688421.02	44709528.37	44730635.72	44711329.34	44732436.69	44753544.04	44774651.39	44755345.01	44776452.36	44797559.71	44818667.06	44839774.41	44860881.75	44881989.10
2017/12	44640803.41	44651357.08	44661910.76	44672464.43	44683018.11	44693571.78	44704125.45	44714679.13	44725232.80	44735786.48	44746340.15	44756893.83	44767447.50	44778001.18	44788554.85	44799108.53	44809662.20	44820215.88

	Number of Retiring	New Hiring	Gapped Positions	Operational/ Gapping Savings	VAC,STAT, TOR per shift	Absentee per Shift due to ADS/Sick/W CB	Projected Staffing Pool Costs	Absentee per Shift due to Other (BRV,FLA etc)	Projected Overtime Coverage Costs	Anticipated Regular Salary	MONTHLY Total
2017/01	2		-8	$ (84,429.40)	14	4	$ 80,827.45	0.8	45090.4691	$ 3,405,400	$ 3,446,888.38
2017/02	0		-8	$ (84,429.40)	15	5	$ 161,654.91	0.6	37134.657	$ 3,530,143	$ 3,644,502.99
2017/03	0		-8	$ (84,429.40)	15	4	$ 121,241.18	0.5	29357.8601	$ 3,530,143	$ 3,596,312.47
2017/04	2		-10	$ (105,536.75)	15	3	$ 80,827.45	0.4	22800.4882	$ 3,530,143	$ 3,528,234.02
2017/05	5	11	-4	$ (42,214.70)	14	4	$ 121,241.18	0.3	18648.1195	$ 3,530,143	$ 3,627,817.43
2017/06	1		-5	$ (52,768.37)	15	4	$ 121,241.18	0.3	19950.644	$ 5,276,542	$ 5,364,965.88
2017/07	1		-6	$ (63,322.05)	17	3	$ 121,241.18	0.4	21501.0787	$ 3,530,143	$ 3,609,563.04
2017/08	1		-7	$ (73,875.72)	17	3	$ 121,241.18	0.3	18661.148	$ 3,530,143	$ 3,596,169.43
2017/09	4		-11	$ (116,090.42)	16	3	$ 121,241.18	0.4	25125.4225	$ 3,577,643	$ 3,607,919.01
2017/10	0		-11	$ (116,090.42)	15	3	$ 80,827.45	0.4	21881.4082	$ 3,577,643	$ 3,564,261.27
2017/11	0		-11	$ (116,090.42)	14	3	$ 40,413.73	0.4	22569.2988	$ 3,577,643	$ 3,524,535.43
2017/12	0		-11	$ (116,090.42)	15	3	$ 80,827.45	0.4	23273.6162	$ 3,577,643	$ 3,565,653.48
										Grand Total	$44,676,822.83

FIGURE 13. Optimal hiring schedule using 2017 projections.

	Number of Retiring	New Hiring	Gapped Positions	Operational/ Gapping Savings	VAC,STAT, TOR per shift	Absentee per Shift due to ADS/Sick/W CB	Projected Staffing Pool Costs	Absentee per Shift due to Other (BRV,FLA etc)	Projected Overtime Coverage Costs	Anticipated Regular Salary	MONTHLY Total
2017/01	2		-8	$ (84,429.40)	14	4	$ 80,827.45	0.8	45090.4691	$ 3,405,400	$ 3,446,888.38
2017/02	0		-8	$ (84,429.40)	15	5	$ 161,654.91	0.6	37134.657	$ 3,530,143	$ 3,644,502.99
2017/03	0		-8	$ (84,429.40)	15	4	$ 121,241.18	0.5	29357.8601	$ 3,530,143	$ 3,596,312.47
2017/04	2		-10	$ (105,536.75)	15	3	$ 80,827.45	0.4	22800.4882	$ 3,530,143	$ 3,528,234.02
2017/05	5	11	-4	$ (42,214.70)	14	4	$ 121,241.18	0.3	18648.1195	$ 3,530,143	$ 3,627,817.43
2017/06	1		-5	$ (52,768.37)	15	4	$ 121,241.18	0.3	19950.644	$ 5,276,542	$ 5,364,965.88
2017/07	1		-6	$ (63,322.05)	17	3	$ 121,241.18	0.4	21501.0787	$ 3,530,143	$ 3,609,563.04
2017/08	1		-7	$ (73,875.72)	17	3	$ 121,241.18	0.3	18661.148	$ 3,530,143	$ 3,596,169.43
2017/09	4		-11	$ (116,090.42)	16	3	$ 121,241.18	0.4	25125.4225	$ 3,577,643	$ 3,607,919.01
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2017/12	0		-11	$ (116,090.42)	15	3	$ 80,827.45	0.4	23273.6162	$ 3,577,643	$ 3,565,653.48
										Grand Total	$44,676,822.83

FIGURE 14. Two-year optimal hiring schedule using 2017 and 2018.

5.3. **Sensitivity Analysis.** We performed a sensitivity analysis on our model to analyze how different absence hours and whether a different bound for the amount of new firefighters affect our optimal schedule from last section.

We first studied whether different absence hours affect the optimal schedule. We use two different methods to perform our analysis: increase or decrease the incapacity absence hours by percentage, and use the absence hours produced by another model.

We increase the incapacity absence hours forecast by SARIMA$(3,1,4)\times(0,1,0,12)$ by 10% and keep other inputs the same as previous section and then rerun our scheduling model. We do the same for increasing 20% and decreasing 10% and 20%. We only do this experiment up to 20% but not for 30% and 40% because in those cases, the total absence hours may too small or too large. The results are shown in Table 2.

Percentage of Original Forecast	**80%**	**90%**	**100%**	**110%**	**120%**
Optimal Budget	44,393,927	44,555,582	44,676,823	44,838,478	45,000,133
Optimal Schedule	Hire 11 firefighters in May				

TABLE 2. Sensitivity Analysis of Effects of Percentage Change in Forecast Result

From our result, increasing or decreasing the absence hours slightly will change the optimal budget but the optimal hiring schedule remains the same. Even if we expand our analysis to 30% and 40%, the result is still the same. We believe that increasing or decreasing the incapacity absence hours by percentage but not change the trend will not affect the optimal hiring schedule.

Next, we use the incapacity absence hours forecast by other models instead the incapacity absence hours we use in previous scheduling model. We will use the results of SARIMA$(2,1,1)\times(2,0,0,12)$, SARIMA$(3,1,0)\times(2,0,0,12)$, SARIMA$(4,1,0)\times(1,1,0,12)$ from last section as well as the simulation model in the previous project of Surrey Fire Department. The results are shown in Table 3. The result suggest that different model will not affect the optimal schedule.

Model	**SARIMA** $(3,1,4)\times$ $(0,1,0,12)$	**SARIMA** $(2,1,1)\times$ $(2,0,0,12)$	**SARIMA** $(3,1,0)\times$ $(2,0,0,12)$	**SARIMA** $(4,1,0)\times$ $(1,1,0,12)$	**Simulation Model**
Optimal Budget	44,676,823	44,434,340	44,878,891	44,515,168	44,636,409
Optimal Schedule	Hire 11 firefighters in May				

TABLE 3. Sensitivity Analysis of Effects of Different Model used in Forecasting

In previous experiments, we restricted the minimum number of new hires to 8. We also evaluated how the optimal schedule change if we change this lower bound. We use the incapacity absence hours forecast by SARIMA$(3,1,4)\times(0,1,0,12)$ for this analysis. The results are shown in Table 4.

Lower Bound	Optimal Schedule	Optimal Budget
1	Hire 3 firefighters in May.	44,526,766
2	Hire 3 firefighters in May.	44,526,766
4	Hire 4 firefighters in June.	44,609,395
6	Hire 7 firefighters in April.	44,615,050
8	Hire 11 firefighters in May.	44,676,823
10	Hire 11 firefighters in May.	44,676,823
12	Hire 12 firefighters in June.	44,755,849
14	Hire 15 firefighters in April.	44,768,708

TABLE 4. Sensitivity Analysis of Effects of Lower Bound of Hirings

From the results, when the lower bound of the amount of hires changes, the optimal schedule and optimal budget will be change. Furthermore, when the lower bound decreases, the optimal budget also decreases while the other inputs keep the same. On one hand, the sensitivity analysis suggests that trying to reduce the lower bound may be a route to cost savings. On the other hand, this will increase the gapped positions that need more overtimes to fill in. More overtime is bad for the health of the firefighters, and may cause more complaints. Further, we saw that a longer range (2 year) planning horizon requires hiring more firefighters than the minimum, and earlier in the year, which suggests the solutions with the minimum in place in fact make more sense. Thus, although decreasing the lower bound suggests further savings are available, we are sceptical.

5.4. **Validation of Hiring Scheduling Model.** Since the Surrey Fire Department provided their 2017 data from January to September, we replace parts of our forecast incapacity absence hours with these actual data for the input of our hiring scheduling model. We reran the scheduling model with this data, the results are shown in Figure 15. The optimal schedule is the same as the optimal schedule in Section 5.2.

6. CONCLUSION

In this project, we studied the effects of season and age on firefighter incapacity absences. We also divided the incapacity absences into long-term and short-term absences, and examined them separately. We believe there are seasonal effects on the firefighter incapacity absences but it is hard to capture all the variables that will affect the absences. Thus we used a time series forecast model (SARIMA) to forecast incapacity absences because the time-series approach has the advantage of learning purely from the data, rather than needing to posit mechanisms. After we received the data for the first three quarters of 2017, we validated the model with this data. We believe that as more historical data becomes available, the result from the SARIMA model will be more accurate. We also optimize the hiring schedule of the Fire Department. Although our optimized results do not make a big difference with their current schedule in 2017, it may be helpful for their future planning.

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2017/02	44717236.56	44752499.52	44828176.22	44742198.00	44817874.69	44893551.39	44969228.08	44883249.86	44999340.28	45034603.25	45110279.94	45064715.46	45180805.88	45296896.30	45372572.99	45407835.96	45523926.38	45640016.80
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2017/09	44692022.96	44734237.66	44776452.36	44737839.60	44780054.30	44781855.27	44824069.97	44785457.21	44827671.91	44869886.61	44912101.31	44913902.28	44956116.98	44957917.95	45000132.65	45042347.35	45084562.04	45126776.74
2017/10	44688421.02	44720082.04	44751743.07	44742990.36	44774651.39	44765898.68	44797559.71	44788807.00	44820468.03	44852129.05	44883790.07	44875037.37	44906698.40	44897945.69	44929606.72	44961267.74	44992928.76	45024589.79
2017/11	44644405.35	44665512.70	44686620.05	44667313.67	44688421.02	44709528.37	44730635.72	44711329.34	44732436.69	44753544.04	44774651.39	44755345.01	44776452.36	44797559.71	44818667.06	44839774.41	44860881.75	44881989.10
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2017/02	0		-8	$ (84,429.40)	15	5	$ 161,654.91	0.6	37134.657	$ 3,530,143	$ 3,644,502.99
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2017/04	2		-10	$ (105,536.75)	15	3	$ 80,827.45	0.4	22800.4882	$ 3,530,143	$ 3,528,234.02
2017/05	5	11	-4	$ (42,214.70)	14	4	$ 121,241.18	0.3	18648.1195	$ 3,530,143	$ 3,627,817.43
2017/06	1		-5	$ (52,768.37)	15	4	$ 121,241.18	0.3	19950.644	$ 5,276,542	$ 5,364,965.88
2017/07	1		-6	$ (63,322.05)	17	3	$ 121,241.18	0.4	21501.0787	$ 3,530,143	$ 3,609,563.04
2017/08	1		-7	$ (73,875.72)	17	3	$ 121,241.18	0.3	18661.148	$ 3,530,143	$ 3,596,169.43
2017/09	4		-11	$ (116,090.42)	16	3	$ 121,241.18	0.4	25125.4225	$ 3,577,643	$ 3,607,919.01
2017/10	0		-11	$ (116,090.42)	15	3	$ 80,827.45	0.4	21881.4082	$ 3,577,643	$ 3,564,261.27
2017/11	0		-11	$ (116,090.42)	14	3	$ 40,413.73	0.4	22569.2988	$ 3,577,643	$ 3,524,535.43
2017/12	0		-11	$ (116,090.42)	15	3	$ 80,827.45	0.4	23273.6162	$ 3,577,643	$ 3,565,653.48
										Grand Total	$44,676,822.83

FIGURE 15. Optimal hiring schedule for 2017 using actual data for January through September.

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