Due: Friday, October 5th (in class)

Reading

From the textbook, Sections 3.3, 4.1, 4.2 and the beginning of Section 4.3.

Chapters 3 and 4 of the AMPL book, available at:

http://ampl.com/resources/the-ampl-book/chapter-downloads/

(Optional) Chapter 2 of Applegate, Bixby, Chvátal and Cook, which shows several interesting problems that can be modelled as a TSP, and hence as an integer program.

Problems for Math 408 and Math 708

1. Consider the personal knapsack you made in the first homework assignment.

   a. Construct a primal (lower) bound for the optimum using the following greedy algorithm: begin with the feasible point $\vec{x} = 0$. For each i from 1 to 9 in turn, see if the point remains feasible if x_i is set to 1. If it is, then set keep $x_i = 1$. Otherwise, reset to $x_i = 0$.

   b. Construct a dual (upper) bound for the optimum by solving the LP relaxation of the problem.

   c. Compare the two bounds to the optimum you found in the first homework assignment, by writing a simple inequality of the form $l \leq opt \leq u$.

2. Exercise 20.2 from the AMPL book. Note that `.mod` and `.dat` for the relevant examples in Chapter 4 are available at:

https://ampl.com/resources/the-ampl-book/example-files/

Please submit your answer to problem 2 directly to the teaching assistant by e-mail (`xla97` at `sfu` dot `ca`). All file names should begin: `math_408_1187_hw2_name_q2` where `name` is your family name. Submit the relevant .dat, .mod and .pdf files showing your output in a single e-mail. If you would like to submit additional questions via e-mail, you may do so, but only if they are *typeset*. (Do **not** include documents that are produced by a scanner.) If you wish to do this, include your remaining answers either in a single .pdf file named: `math_408_1187_hw2_name_qall`, or with one .pdf file per question names `math_408_1187_hw2_name_q1` (or q2, q4, q5, q6, q7, q8 as appropriate). Here again, `name` is substituted with your own family name.

3. Textbook Exercise 1.4.

4. Textbook Exercise 2.1.

5. Give an example of a $\{-1, 0, 1\}$ matrix A and an integer vector b such that the set $\{Ax \leq b \mid x \in \mathbb{R}^n\}$ is an integer polytope, but A is not totally unimodular.

Additional Problems for Math 708

6. Consider the `crucipixel` game of textbook exercise 2.30.

   a. Formulate the particular 10×10 example given in the problem as a 0-1 linear program.

   b. Solve the problem with AMPL, submitting files as in Problem 2.

   c. Extend your solution to an arbitrary $m \times n$ `crucipixel` game.

7. Textbook Exercise 4.9.

8. Consider modelling a scheduling problem where machine can be switched on at most k times, with discrete

time segments indexed by t. This problem can be modelled using variables y_t representing whether the machine is on during period t, and z_t which representing whether the switching on happened during period t. This can be formulated via the following inequalities:

$$y_t - y_{t-1} \leq z_t \leq y_t \text{ for all } t; \qquad \sum_t z_t \leq k; \qquad 0 \leq y_t, z_t \leq 1 \text{ for all } t.$$

Show that the matrix encoding these constraints is totally unimodular.

Late Policy

Assignments should be submitted on time. Late assignments must be typeset and submitted by e-mail directly to the TA. One point (out of the assignment total of 40 or 60) will be deducted every 2 hours starting at 9:30 a.m. on the deadline day.

Graduate Student Projects

Math 708 students will give presentations surveying additional topics in integer programming. These presentations will take place in the final full week of class, on Monday, November 26th, Wednesday, November 28th or Friday, November 30th. Presentations will last for 20 minutes, followed by 5 minutes for questions.

Possible sources for topics include sections of the textbook not covered in class, the book of Bertsimas and Weismantel [1] and the book of de Lorea, Hemmecke and Köppe [2], Other topics may be possible, please see the instructor if you have something else in mind. It may be helpful to discuss with your advisor which topics are relevant to your research. Please sign-up for a date and topic. First-come, first-served.

Students who are interested in doing a more extensive and in-depth presentation may consider presenting a 45 minute talk in the Operations Research Seminar on Thursdays (November 29th or December 6th) at 3:30 subject to availability and consent of the instructor.

References

[1] Dimitris Bertsimas and Robert Weismantel. *Optimization over Integers*. Dynamic Ideas, Belmont, MA, 2005.

[2] Jesús A. De Loera, Raymond Hemmecke, and Matthias Köppe. *Algebraic and geometric ideas in the theory of discrete optimization*, volume 14 of *MOS-SIAM Series on Optimization*. Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA; Mathematical Optimization Society, Philadelphia, PA, 2013.