Due: Friday, November 13th (noon)

Reading

From the textbook: Section 4.4.4.

Chapter 7 through Section 7.2.1.

Sections 2.7 and 7.4, briefly skim 7.5.

Chapter 3 of Applegate, Bixby, Chvátal and Cook, which tells a little bit about the development of the ideas that we are covering.

Problems for Math 408 and Math 708

All problems to be submitted via Canvas. Please submit a single file names `hw4.pdf` containing all your written work, along with files `hw4.dat` and `hw4.mod` for the AMPL question (question 4); for this exercise separate files for part (b) would be helpful, which you can call `hw4b.dat` and `hw4b.mod`. Please make sure to write your name on the first page of `hw4.pdf` and in the comments of `AMPL` files.

1. Consider the personal knapsack problem you made in the first homework assignment.

a. Let S be the set of 0-1 points feasible for your knapsack problem. What dimension is the face of conv(S) defined by your personal knapsack inequality?

b. Use your personal knapsack inequality to derive 3 minimal cover inequalities for conv(S).

2. Consider the polytope in $\mathbb{R}^4$ generated by taking the convex hull of the points $(\pm 1, 0, 0, 0)$, $(0, \pm 1, 0, 0)$, $(0, 0, \pm 1, 0)$, and $(0, 0, 0, \pm 1)$. Describe all of its faces. How many are there in total?

3. Consider the *cut polytope* of Example 3.36 in the text. Show that it is full-dimensional for $n = 2, 3, 4$.

4. Exercise 20.4 (a) and (b) from the AMPL book, available at:

`http://ampl.com/resources/the-ampl-book/chapter-downloads/`.

5. Consider the cone:

$$C = \{(x, y) \in \mathbb{R}^2 \mid 2x \leq 5y,\ 2y \leq 5x\}.$$

Find the minimal Hilbert basis for $C \cap \mathbb{Z}^2$. That is, find the minimal set of integer vectors such that every point in C can be expressed as a non-negative integer combination of those vectors.

Additional Problems for Math 708

6. Consider the stable set formulation from the previous assignment, where inequalities are associated to edges. Take the graph W_k which consists of a k-cycle and a single vertex v_0 attached to each vertex of the cycle. Such graphs are sometimes called *wheels*. The 5-cycle inequality is valid for the 5-wheel, or *pentastar*, as Chrysler calls it.

a. What is the dimension of the stable set polytope of the 5-wheel?

b. What is the dimension of the face of this polytope induced by the 5-cycle inequality?

c. ~~Lift this face to a facet by adding a term representing the variable x_{v_0} to the inequality.~~

7. Exercise 20.4 (c) and (d) from the AMPL book. Submissions should follow the format of problem 4, but file names `hw4-7.mod` and `hw4-7.dat`. The volume limit is not specified in part (d), take it to be 12.

8. A system of linear inequalities $\{A\mathbf{x} \leq \mathbf{b}\}$ is totally dual integral (TDI) if for all $\mathbf{c} \in \mathbb{Z}^n$ such that $\{\max \mathbf{c}^t\mathbf{x} \mid A\mathbf{x} \leq \mathbf{b}\}$ has a finite optimum value, the dual linear program $\{\min \mathbf{b}^t\mathbf{y} \mid A^t\mathbf{y} = \mathbf{c}, \mathbf{y} \geq \mathbf{0}\}$ has an integer optimum.

Show that the system $\{(x,y) \in \mathbb{R}^2 \mid x+y \leq 0, x-y \leq 0\}$ is not TDI, but that if we add the redundant inequality $x \leq 0$, the system becomes TDI.

Comments: In fact, if $\{A\mathbf{x} \leq \mathbf{b}\}$ is TDI, then P is the convex hull of S. If A is TUM, then $\{A\mathbf{x} \leq \mathbf{b}\}$ is TDI for *any* $\mathbf{b} \in \mathbb{Z}^n$.

Tentative schedule of graduate presentations

Each graduate student will present a brief introductory lecture on an additional topic in integer programming. This should contain substantial mathematical content and be understandable to the undergraduate students. The talks will be 20 minutes, followed by a 5 minute question period. Overheads will be submitted as part of the grading.

These talks will take place in December. The tentative schedule and topics are as follows:

Wednesday, December 2nd **Danielle Rogers**, on Semi-definite Programming and Stable Set.

Friday, December 4th (early) **Einar Gabbassov**, on Quantum Annealing for Combinatorial Optimization.

Friday, December 4th (late) **Alborz Namazi**, on Computer Codes for Integer Programming.

Monday, December 7th (early) **Sajeththa Thavayogarajah**, on the Quadratic Assignment Problem.

Monday, December 7th (early) **Brett Wiens**, on Optimization on Gray Codes.

Please let me know about any possible errors or adjustments in this schedule.