

Due: Monday, November 8th (11:59 p.m. PT.)

Reading

For Wednesday, October 27th, Section 4.8. Sections 4.9, 4.10 and 4.11 are also worth skimming.

For Friday, October 29th, Sections 5.1 and 5.2.1.

For Monday, November 1st, Section 5.2.2.

For Wednesday, November 3rd, Section 5.2.3.

For Friday, November 5th, Section 5.2.4. Section 5.2.5 can be skimmed.

For Monday, November 8th, Sections 5.3.1 and 5.3.2.

Assignment exercises to hand in

1. Is your personal Markov chain (from Assignment 3) time reversible?
2. Take the digits of your student id, in order: $a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8$ and a_9 , and let $N = \sum_{i=1}^9 a_i$. Your *personal branching process* produces 0 offspring with probability $(a_1)/N$, 1 offspring with probability $(a_2 + a_3 + a_4 + a_5)/N$, and 2 offspring with probability $(a_6 + a_7 + a_8 + a_9)/N$. Starting from one individual ($X_0 = 1$), what is the expected number of individuals in the n^{th} generation?
3. What is the probability the population will die out on the first generation (i.e. $P\{X_1 = 0\}$)? What is the probability the population will die out by the second generation (i.e. $P\{X_2 = 0\}$)?
4. What is the probability the population will ever die out (i.e. $P\{X_n = 0\}$ for some $n \geq 0$)?
5. Using the CBC Snakes and Ladders game from:
<https://www.cbc.ca/kids/games/play/snakes-and-ladders>, describe the full transition matrix for the associated Markov chain. Do not construct states corresponding to squares that are at the base of a ladder or tail of a snake.
The Markov transition matrix you represent is for the movement of a single player. Use the convention from class that each single roll of the dice constitutes a move: ignore the “roll again” following a 6 rule that is implemented on-line. Also, if a roll takes you past the end state (e.g. rolling a 5 from square 97), it will bring you to the end state.
6. What is the submatrix corresponding to transient states? Compute the mean time spent in the transient states. What are the 3 most frequently visited states? (This question will require the use of software, such as MATLAB.)
7. Chapter 4, Exercise 35
8. Chapter 4, Exercise 50
9. Chapter 5, Exercise 4
10. Chapter 5, Exercise 6
11. Chapter 5, Exercise 9
12. Chapter 5, Exercise 14
13. Chapter 5, Exercise 15
14. Chapter 5, Exercise 20