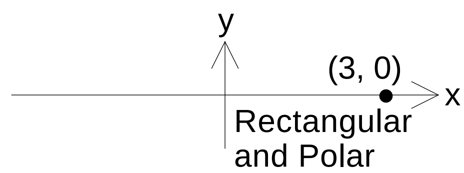
MATHEMATICS 151

Assignment 34 Page 552

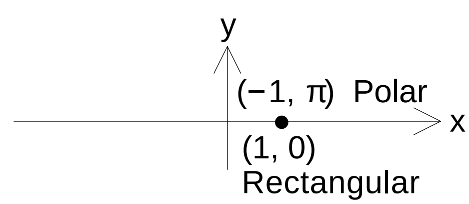


For Exercise 2

2. $x = 3\cos 0 = 3$ and $y = 3\sin 0 = 0$.

Other polar coordinates: $(3, 2\pi)$, $(-3, \pi)$.
There are infinitely many more:
$(3, 2n\pi)$ and $(-3, (2n+1)\pi)$, n any integer.

See graph to the right.

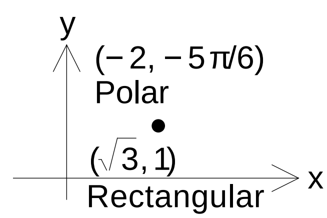


For Exercise 6

6. $x = (-1)\cos\pi = 1$ and $y = (-1)\sin\pi = 0$.

Other polar coordinates: $(1, 0)$, $(-1, 3\pi)$.
There are infinitely many more:
$(1, 2n\pi)$ and $(-1, (2n+1)\pi)$, n any integer.

See graph to the right.



For Exercise 12

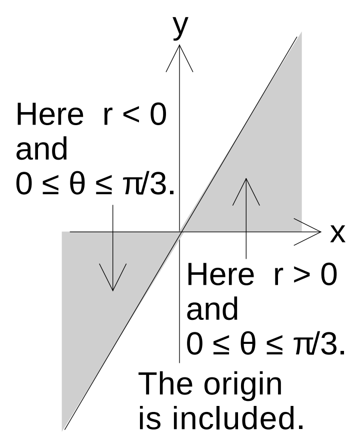
12. $x = (-2)\cos\left(-5\pi/6\right) = \sqrt{3}$
and $y = (-2)\sin\left(-5\pi/6\right) = 1$.

See graph below and to the right.

16. If $(x, y) = (3, 4)$ and $r > 0$, $r = \sqrt{3^2 + 4^2} = 5$.
If $0 \le \theta < 2\pi$ then $\tan\theta = 4/3$, so $\theta = \tan^{-1}(4/3)$
since $(3, 4)$ is in the first quadrant and $r > 0$. Polar coordinates: $(5, \tan^{-1}(4/3))$.

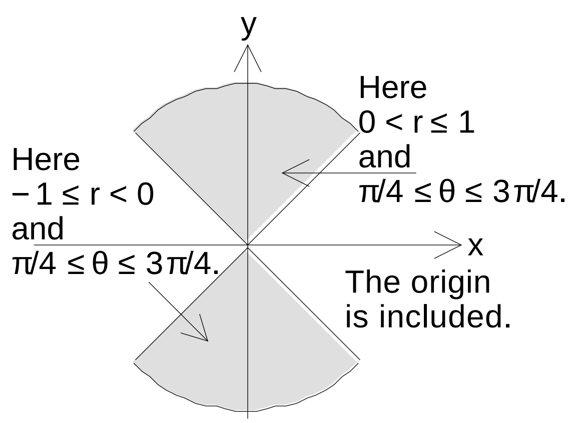
18. $0 \le \theta \le \pi/3$.

See graph below.



22. $-1 \le r \le 1$, $\pi/4 \le \theta \le 3\pi/4$.

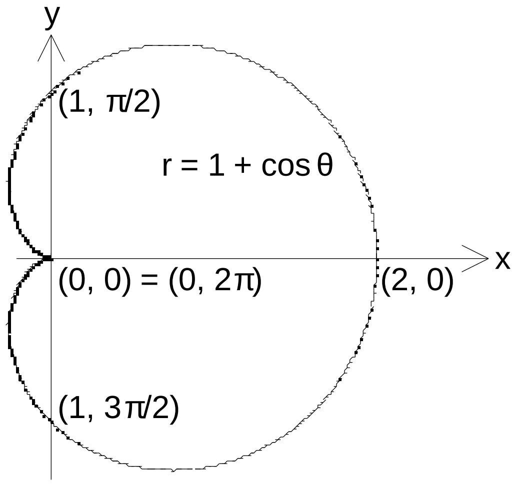
See graph below.



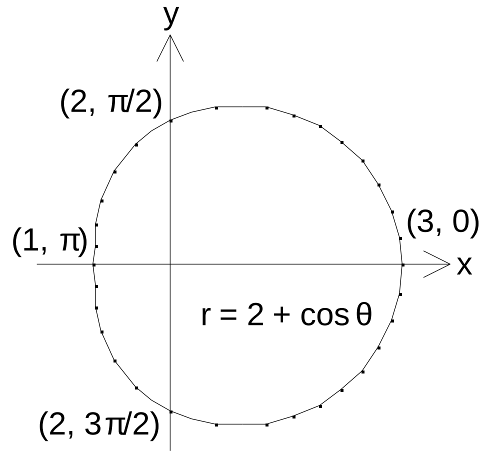
24. $D = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \sqrt{(r_1 \cos\theta_1 - r_2\cos\theta_2)^2 + (r_1\sin\theta_1) - r_2\sin\theta_2)^2} =$
$= \sqrt{r_1^2 + r_2^2 - 2r_1r_2(\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2)} = \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_1 - \theta_2)}$.

36. $x^2 - y^2 = 1$, so $r^2(\cos^2\theta - \sin^2\theta) = 1$, $r^2\cos(2\theta) = 1$, $r^2 = \sec(2\theta)$, and $r = \pm\sqrt{\sec(2\theta)}$.

44. $r = 1 + \cos\theta$.
See graph below.



50. $r = 2 + \cos\theta$.
See graph below.

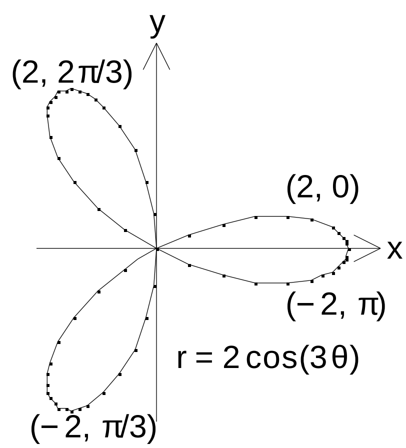


52. $r = 2\cos(3\theta)$. See graph below and to the right.

66. $r = \ln\theta$ so $\frac{dr}{d\theta} = \frac{1}{\theta}$, and

$$\frac{dy}{dx} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta} = \frac{\frac{\sin\theta}{\theta} + \cos\theta\ln\theta}{\frac{\cos\theta}{\theta} - \sin\theta\ln\theta}.$$

At $\theta = e$, $\frac{dy}{dx} = \frac{\frac{\sin e}{e} + \cos e\ln e}{\frac{\cos e}{e} - \sin e\ln e} = \frac{\sin e + e\cos e}{\cos e - e\sin e}$.



For Exercise 52

70. $r = \cos\theta + \sin\theta$ so $\frac{dr}{d\theta} = -\sin\theta + \cos\theta$.

For a horizontal tangent, $\frac{dr}{d\theta}\sin\theta + r\cos\theta = 0$.
Hence $-\sin^2\theta + \sin\theta\cos\theta + \cos^2\theta + \cos\theta\sin\theta = 0$, or $\cos(2\theta) + \sin(2\theta) = 0$, and $\tan(2\theta) = -1$, so that $\theta = -\frac{\pi}{8} + \frac{n}{2}\pi$, n an integer.
The corresponding points are the points with these values of θ and at which $r = \cos\theta + \sin\theta$.

For a vertical tangent, $\frac{dr}{d\theta}\cos\theta - r\sin\theta = 0$.
Hence $-\cos\theta\sin\theta + \cos^2\theta - \cos\theta\sin\theta - \sin^2\theta = 0$, or $\cos(2\theta) - \sin(2\theta) = 0$, and $\tan(2\theta) = 1$, so that $\theta = \frac{\pi}{8} + \frac{n}{2}\pi$, n an integer.
The corresponding points are the points with these values of θ and at which $r = \cos\theta + \sin\theta$.

It is possible (using the double angle formulas to express $\cos(\pi/4)$ in terms of $\cos(\pi/8)$ or $\sin(\pi/8)$) to find the values of $\cos(\pi/8)$ and $\sin(\pi/8)$ and obtain closed form expressions for the values of r, using radicals.