MATHEMATICS 151

2. $\lim_{x \to 1} \frac{x^2 + 3x - 4}{x - 1} = \lim_{x \to 1} \frac{2x + 3}{1} = 5.$

Or avoid L'Hospital's Rule with $\lim_{x \to 1} \frac{x^2 + 3x - 4}{x - 1} = \lim_{x \to 1} \frac{(x-1)(x+4)}{x - 1} = \lim_{x \to 1} (x + 4) = 5.$

10. $\lim_{x \to 3\pi/2} \frac{\cos x}{x - \frac{3\pi}{2}} = \lim_{x \to 3\pi/2} \frac{-\sin x}{1} = 1.$

Or avoid L'Hospital's Rule with $\lim_{x \to 3\pi/2} \frac{\cos x}{x - \frac{3\pi}{2}} = \lim_{x \to 3\pi/2} \frac{\sin\left(x - \frac{3\pi}{2}\right)}{x - \frac{3\pi}{2}} = 1.$

16. $\lim_{x \to 0} \frac{6^x - 2^x}{x} = \lim_{x \to 0} \frac{6^x \ln 6 - 2^x \ln 2}{1} = \ln 6 - \ln 2 = \ln \frac{6}{2} = \ln 3.$

Or avoid L'Hospital's Rule in the following way.

Let $f(x) = 6^x - 2^x$. Then $f'(x) = 6^x \ln 6 - 2^x \ln 2$, so $f'(0) = \ln 6 - \ln 2 = \ln \frac{6}{2} = \ln 3.$

But $\lim_{x \to 0} \frac{6^x - 2^x}{x} = \lim_{x \to 0} \frac{(6^x - 2^x) - (6^0 - 2^0)}{x - 0} = f'(0)$, from the definition of the derivative.

22. $\lim_{x \to 0} \frac{\sin x - x}{x^3} = \lim_{x \to 0} \frac{\cos x - 1}{3x^2} = \lim_{x \to 0} \frac{-\sin x}{6x} = \lim_{x \to 0} \frac{-\cos x}{6} = -\frac{1}{6}.$

30. $\lim_{x \to 0} \frac{\sin(mx)}{\sin(nx)} = \lim_{x \to 0} \frac{m\cos(mx)}{n\cos(nx)} = \frac{m}{n}.$

Or avoid L'Hospital's Rule by writing $\lim_{x \to 0} \frac{\sin(mx)}{\sin(nx)} = \lim_{x \to 0} \frac{\frac{\sin(mx)}{mx}}{\frac{\sin(nx)}{nx}} \cdot \frac{m}{n} = \frac{1}{1} \cdot \frac{m}{n} = \frac{m}{n}.$

44. $\lim_{x \to 0^+} \sqrt{x} \sec x = 0\ 1 = 0.$ L'Hospital's Rule is inapplicable.

48. $\lim_{x \to 0} (\csc x - \cot x) = \lim_{x \to 0} \frac{1 - \cos x}{\sin x} = \lim_{x \to 0} \frac{\sin x}{\cos x} = 0.$

56. Let $y = (\sin x)^{(\tan x)}$, $0 < x < \pi/2$. Then $\ln y = (\tan x)(\ln(\sin x))$.

So $\lim_{x \to 0^+} \ln y = \lim_{x \to 0^+} \frac{\ln(\sin x)}{\cot x} = \lim_{x \to 0^+} \frac{\frac{\cos x}{\sin x}}{-\csc^2 x} = \lim_{x \to 0^+} (-\sin x \cos x) = 0.$

Hence $\lim_{x \to 0^+} y = e^0 = 1.$

78. $\lim_{x \to +\infty} \frac{\ln x}{x^p} = \lim_{x \to +\infty} \frac{x^{-1}}{px^{p-1}} = \lim_{x \to +\infty} \frac{1}{px^p} = 0$, if $p > 0$.