

## APMA 990 Wavelets — Problem Set 1

Report by Monday, January 28, 2002

**Problem 1** Calculate the Fourier series expansions of the following functions, and verify the symmetry properties of the coefficients:

- (i)  $f$  has period 2, and  $f(t) = |t|$  for  $|t| < 1$ .
- (ii)  $f$  has period  $a$ , and  $f(t) = t/a$  for  $0 \leq t < a$ .
- (iii)  $f(t) = |\sin t|$ .
- (iv)  $f(t) = \sin^3 t$ .

**Problem 2** Let  $c_n$  be the Fourier coefficients of the periodic function  $f : t \rightarrow f(t)$ . What are the Fourier coefficients of the "delayed" function  $g : t \rightarrow f(t - t_0)$  ?

Use your result, and the first problem to find the Fourier coefficients of  $f(t) = |\cos t|$ .

**Problem 3** Let  $f$  be periodic with period  $a$ , and denote by  $c_n$  its Fourier coefficients. Then  $f$  is also periodic with period  $2a$ , and corresponding Fourier coefficients  $c'_n$ . What is the relationship between  $c_n$  and  $c'_n$  ? Verify that the two Fourier series are identical.

**Problem 4** Show that for a periodic, twice continuously differentiable function  $f$  we have

$$|c_n(f)| \leq \frac{K}{n^2}.$$

**Problem 5** We define the convolution of two  $2\pi$ -periodic functions  $f$  and  $g$  by

$$(f * g)(s) := h(s) = \frac{1}{2\pi} \int_{-\pi}^{+\pi} f(s - t)g(t)dt.$$

- (a) Show that  $f * g = g * f$ .

- (b) Assuming all quantities in question exist, what are the Fourier coefficients of  $f * g$  in terms of the Fourier coefficients of  $f$  and  $g$  ?
- (c) If both  $f$  and  $g$  are in  $L^2$ , is the convolution  $f * g$  defined ?

**Problem 6** Consider two consecutive discrete Fourier transforms:

$$(y_k) \rightarrow (Y_n), \quad (Y_n) \rightarrow (z_q).$$

Compute  $(z_q)$  as a function of  $(y_k)$ .

**Problem 7** Let  $(x_k)$  and  $(y_k)$  be two complex  $N$ -periodic sequences with

$$x_{N-k} = \bar{x}_k, \quad y_{N-k} = \bar{y}_k,$$

for all  $k$ . Show that the discrete Fourier transforms  $(X_n)$  and  $(Y_n)$  are real, and that they can be computed with a single transform of order  $N$ .

**Problem 8** Compute the successive powers of the Fourier matrix  $\Omega_N$ .

**Problem 9** Calculate the discrete Fourier transform of the vector  $(x_k)$  in  $\mathbb{C}^N$  with  $x_k = k$ , for  $k = 0, \dots, N-1$ .