MATH 155 — Quiz #2

March 10, 1995

Please make sure you received 10 pages (including this cover page) with 5 problems. You have 45 minutes for the exam, and you may attempt the problems in any order. You may use a one-page summary of your notes during this exam. No other help is allowed. Write your answers in the space provided. If you need more space, attach additional pages.

Good Luck!

Name:	
Student number:	

Problem	Maximum	Points received
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

Problem 1 *Definite integrals, rates of change.* The doors to a *Rolling Stones* concert open at $t = 0$ (t measures the time in hours) two hours prior to the start of the concert (i.e., the concert starts at $t = 2$). The rate of arrival of fans is given by $f(t) = \alpha t e^{-2t}$.

(a) At which time before the concert is the rate of arrival the largest?

(b) Denote by $F(t)$ the number of people in the stadium at time t, assuming that the stadium is empty when the doors open ($F(0) = 0$). Write $F(t)$ as a (definite) integral of $f(t)$.

(c) Evaluate the integral in (b) to obtain a formula for $F(t)$.

(d) For $\alpha = 100000 = 10^5$, how many people are in the stadium

(i) 90 minutes prior to the concert ?

(ii) towards the end of the concert (approximate by $t \to \infty$) ?

x	e^{-x}
0.00	1.00000000
0.50	0.60653066
1.00	0.36787944
1.50	0.22313016
2.00	0.13533528
2.50	0.08208500
3.00	0.04978707
3.50	0.03019738
4.00	0.01831564
4.50	0.01110900
5.00	0.00673795
5.50	0.00408677
6.00	0.00247875
6.50	0.00150344
7.00	0.00091188
7.50	0.00055308
8.00	0.00033546

Solution of Problem 1.

Problem 2 *Binomial and normal distribution.* Assume the probability of passing *Math-155* with a grade of C or higher is 75%.

1. Write down the formula for the exact probability, that in a class of 300 students 240 or more students obtain a grade of C or better.

2. Approximate the binomial distribution by the normal distribution, and obtain an approximate probability of the event that in a class of 300 students 240 or more students obtain a grade of C or better.

The density function g of a *standard normal random variable* is given by $g(s) = \frac{1}{\sqrt{2\pi}}e^{-s^2/2}$. The normal curve area $A(z) = \int_0^z g(s)ds$ is tabulated below:

z	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	.00000	.03983	.07926	.11791	.15542	.19146	.22575	.25804	.28814	.31594
1	.34134	.36433	.38493	.40320	.41924	.43319	.44520	.45543	.46407	.47128
2	.47725	.48214	.48610	.48928	.49180	.49379	.49534	.49653	.49744	.49813
3	.49865	.49903	.49931	.49952	.49966	.49977	.49984	.49989	.49993	.49995

Solution of Problem 2.

Problem 3 *Discrete probabilities.*

(a) How many possibilities are there to have exactly three of the six winning numbers in the 6/49 Lottery correct?

(b) There are 13,983,816 different ways to choose your bet of 6 numbers out of the 49. One bet costs \$1; betting three correct numbers will pay you \$10. For every \$100 in ticket revenue, how much does the Lottery Corporation pay to all the winners with three correct numbers (rounded to the nearest Dollar)?

Solution of Problem 3.

Problem 4 *Areas.* What is the total area between the x-axis and the curve $y = 1 - 2\cos(2x)$ over the interval $0 \leq x \leq \pi$? Show your work. A sketch of the curve will probably be helpful.

Solution of Problem 4.

Problem 5 *Volumes of revolution.* The region enclosed between $y = x^2$ and $y = x + 2$ is revolved about the x-axis. Find the volume of the solid formed. Show your work.

Solution of Problem 5.