MATH 155 — Lecture #4

January 14, 2000

Substitution.

(a) $\int x(x^2-2)^9 dx = \int \frac{1}{2}y^9 dy = \frac{1}{10}y^{10} + C = \frac{1}{20}(x^2-2)^{10} + C$. Use $y = x^2 - 2$.

(b) $\int \sqrt{x-5}dx = \ldots = \frac{2}{3}(x-5)^{3/2} + C$.

(c) $\int t\sin(t^2)dt = \ldots = -\frac{1}{2}\cos(t^2) + C$.

(d) $\int \frac{dx}{x\ln x} = \int \frac{du}{u} = \ln|u| + C = \ln|\ln x| + C$, using $u = \ln x$.

Tables of Integrals. Reading Assignment, Section 6.3.

Partial Fractions.

Example:

$$\begin{aligned} \frac{x+2}{x^2+x} = \frac{x+2}{x(x+1)} &= \frac{A}{x} + \frac{B}{x+1} \\ x+2 &= A(x+1) + Bx = (A+B)x + A \\ 0 &= (A+B-1)x + (A-2) \end{aligned}$$

Therefore,

$$\begin{aligned} A+B &= 1 \\ A &= 2 \end{aligned}$$

Solving this system gives $A = 2$, $B = -1$.

$$\int \frac{x+2}{x^2+x}dx = \int \left(\frac{2}{x} - \frac{1}{x+1}\right)dx = 2\ln|x| - \ln|x+1| + C.$$