MATH 155 — Lecture #8

January 24, 2000

Integration by parts. The product rule for differentiation states that

$$\frac{d}{dx}\left(u(x)v(x)\right) = u'(x)v(x) + u(x)v'(x).$$

Integration by parts is the application of this formula to finding antiderivatives:

$$\int u'(x)v(x)dx = u(x)v(x) - \int u(x)v'(x)dx,$$

or

$$\int u(x)v'(x)dx = u(x)v(x) - \int u'(x)v(x)dx,$$

This rule will be helpful in finding antiderivates if the integral on the right hand side is easier to evaluate than the one on the left hand side of the equation.

Examples (not all exactly the same as in the lecture):

$$\begin{aligned}\int \underbrace{x}_{u(x)}\underbrace{e^{2x}}_{v'(x)}\,dx &= \underbrace{x}_{u(x)}\underbrace{\frac{1}{2}e^{2x}}_{v(x)} - \int \underbrace{1}_{u'(x)}\underbrace{\frac{1}{2}e^{2x}}_{v(x)}\,dx \\ &= \frac{x}{2}e^{2x} - \frac{1}{4}e^{2x} + C.\end{aligned}$$

Repeated application of product rule allows evaluation of integrals of the form

$$\int \underbrace{x^n}_{u(x)}\underbrace{e^{\alpha}}_{v'(x)}\,dx = \underbrace{x^n}_{u(x)}\underbrace{e^{\alpha x}/\alpha}_{v(x)} - \int \underbrace{nx^{n-1}}_{u'(x)}\underbrace{\frac{1}{\alpha}e^{\alpha x}}_{v(x)}\,dx.$$

Sometimes we have to use the function 1 as a factor:

$$\int \ln x\,dx = \int \underbrace{1}_{u'(x)}\underbrace{\ln x}_{v(x)}\,dx = \underbrace{x}_{u(x)}\underbrace{\ln x}_{v(x)} - \int \underbrace{x}_{u(x)}\underbrace{\frac{1}{x}}_{v'(x)}\,dx = x\ln x - x + C.$$

$$\begin{aligned}\int \cos^2 x dx = \int \underbrace{\cos x}_{u(x)}\underbrace{\cos x}_{v'(x)}\,dx &= \underbrace{\cos x}_{u(x)}\underbrace{\sin x}_{v(x)} - \int \underbrace{-\sin x}_{u'(x)}\underbrace{\sin x}_{v(x)}\,dx = \cos x\sin x + \int \sin^2 x\,dx \\ &= \cos x\sin x + \int (1-\cos^2 x)dx = \cos x \sin x + x + \int \cos^2 x\,dx.\end{aligned}$$

Therefore,

$$\int \cos^2 x dx = \frac{1}{2}\left(\cos x\sin x + x\right) + C = \frac{x}{2} + \frac{1}{4}\sin(2x) + C.$$