MATH 416 — PROJECT

Due: Monday, December 2, 2002, **in class**

In this project you will be reproducing some of the figures in the lecture notes, and in the process explore finite difference schemes for solving patial differential equations. Th epage numbers refer to the recently posted complete set of lecture notes for Math 416.

Presentation. Hand in an $8\frac{1}{2}'' \times 11''$ booklet, stapled in the upper left corner. Structure your paper like a report, with title, abstract, overview of the methods used, description of the building blocks. Describe the basic structure of your programs, and include the program listings in an appendix. Present your numerical results in condendsed form, using tables and/or graphs. Include pictures and plots where appropriate. The presentation is an important aspect of the project. It is important to include enough detail to get your message across, while avoiding to just include everything, or describing each line in the code.

Programs. Use double precision (16 digits in Maple). I strongly recommend you use MATLAB.

PART 1 As a warm-up exercise reproduce the calculations and plots for Example 4.1 on Page 50 of the Lecture Notes.

PART 2 Another simple warm-up exercise related to PDEs are pseudospectra of matrices. Reproduce Figure 4.3 (page 53); play around with different "symbols" producing plots of various pseudospectra. Denote by K the matrix with all zeros except for $K_{i,i+1} = 1$. Do your experiments with matrices which are simple polynomials in the matrix K, i.e., $I + K + \frac{1}{2}K^2$.

PART 3 Do the calculations for figure 5.2 on page 72. Experiment with various values for h and λ. **OPTIONAL**: Study the dispersive effects of various finite difference methods by doing the calculations required to produce figures 5.3 and 5.4.

PART 4 We investigate effects of boundary conditions. Perform the calculations required for Figure 5.5 on page 77. As an OPTION, only if you want to do some extra work, produce figures 5.6 and 5.7.

PART 5 OPTIONAL. Experiment with some finite difference methods to solve

$$u_t = -x u_x,$$

on the interval $[0, 1]$. This equation does not require any boundary conditions.

(a) Verify that the exact solution is $u(x, t) = u(xe^{-t}, 0)$.

(b) Try various initial conditions and integrate the PDE until $t = 1$.

Enjoy....